C U R S O S E C O N G R E S O S

6th Iberian Mathematical Meeting

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6th-8th, October, 2016

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Felipe Gago
Manuel Ladra

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Editor's introduction

This volume contains the abstracts of the papers presented at the 6th Iberian Mathematical Meeting held in the Faculty of Mathematics of the University of Santiago de Compostela, Spain, from October 6th to 8th, 2016.

The conference was promoted by the Real Sociedad Matemática Española and the Sociedade Portuguesa de Matemática and following the tradition of these meetings it was structured around three main areas:

- Algebra and Combinatorics,
- Applied Partial Differential Equations in Fluids and Materials,
- Theoretical Informatics/Computer Sciences,

with nearly 60 abstracts, invited, contributed and posters, accepted to be presented as well as 6 plenary conferences

Our warmest thanks to the invited plenary speakers:

Antónia Lópes (Universidade de Lisboa), Salvador Lucas (Universitat Politècnica de València), Pablo Pedregal (Universidad de Castilla-La Mancha), Juan Rafael Sendra (Universidad de Alcalá), Pedro V. Silva (Universidade do Porto) and Ana Leonor Silvestre (Universidade de Lisboa)

We would like to thank all the participants for the attendance and for their valuable contributions.

Finally, the editors gratefully acknowledge the work of the Scientific Committee of the Conference for their large contribution to the success of the event along with the Organizing Committee and the collaborating entities: RSME, SPM, Facultade de Matemáticas da USC, Xunta de Galicia and Concello de Santiago de Compostela.

F. Gago and M. Ladra
Santiago de Compostela, October 2016.

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Plenary Conferences

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A Component Algebra for Heterogeneous Networks of Timed Systems

Antónia Lopes[1]

The systems that are now operating in cyberspace are best modelled as networks of 'machines', where each machine performs local computations and can be interconnected at run time to other machines to achieve some goal. Because of the distributed nature of such networks, it does not make sense to assume that the nodes of the networks at which machines execute have the same clock granularity.

In this talk I will present a component algebra and an associated logic for these type of systems. The components of the algebra are asynchronous networks of processes that abstract the behaviour of machines that execute according to the clock granularity of the network node in which they are placed and communicate asynchronously with machines at other nodes. The main novelty of this theory is that not all network nodes need to have the same clock granularity. I will discuss conditions under which it can be guaranteed, a priori, that any interconnections generated at run time through dynamic binding of machines with different clock granularities leads to a consistent orchestration of the whole system. I will provide an automata-theoretic view of these results and discuss which logics can support specifications for this component algebra.

This is joint work with J.L. Fiadeiro (Royal Holloway, University of London).

Keywords: component algebra, timed systems, clock granularity

MSC 2010: 68N30, 68M14

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Navier-Stokes flows around moving obstacles

Ana Leonor Silvestre[1]

Consider a rigid body $\mathcal{S} \subset \mathbb{R}^3$ moving through a viscous incompressible fluid which fills the exterior domain $\Omega := \mathbb{R}^3 \setminus \mathcal{S}$, and the steady state regime of the system body-fluid, as seen by an observer attached to $\mathcal{S}$. The equations modeling this mechanical system is

$$-\nabla \cdot \sigma(v,p) + \varrho(v-V)\cdot\nabla v + \varrho\omega \times v = \varrho f \quad \text{in } \Omega \tag{1}$$

$$\nabla \cdot v = 0 \quad \text{in } \Omega \tag{2}$$

$$v = V + v_* \quad \text{on } \partial\Omega \tag{3}$$

$$\lim_{|x|\to\infty} v(x) = 0 \tag{4}$$

where the quantities $v = v(x)$ and $p = p(x)$ represent, respectively, the unknown velocity field and the pressure of the liquid, $V(x) = \xi + \omega \times x$ represents the velocity of the solid, v_* is a distribution of velocities on the surface of $\mathcal{S}$ and f is an external force. The positive constant ϱ is the density of the fluid and $\sigma(v,p)$ is the Cauchy stress tensor, defined by $\sigma(v,p) := 2\nu D(v) - pI_3$, with ν the viscosity of the fluid, I_3 the 3×3 identity matrix and $D(v) := \frac{1}{2}\left((\nabla v) + (\nabla v)^\top\right)$.

Along with classical mathematical problems such as existence, uniqueness and regularity of solutions to system (1)–(4), we are interested in a rigorous justification of physical properties such as

(P1) the validity of the *energy equation*

$$2\nu\int_\Omega |D(v)|^2\,dx - \int_{\partial\Omega}\big(\sigma(v,p)n\big)\cdot(V+v_*)\,d\gamma = \varrho\int_\Omega f\cdot v\,dx - \frac{\varrho}{2}\int_{\partial\Omega}(v_*\cdot n)|V+v_*|^2\,d\gamma, \tag{5}$$

(P2) the anisotropic structure of the flow, characterized by the formation of a *wake region* behind the solid whose "width" apparently depends on the angle between ω and ξ.

We will see what is the role of the generalized Oseen fundamental solution and the so-called Landau solution in the derivation of precise information about the asymptotic behavior of v and p at infinity, which, in turn, is crucial to obtain the summability properties of solutions and allows to justify (5) rigorously (due to the non-linear term

in (1) some smallness conditions are also needed). The Oseen fundamental solution justifies the expected non-uniform behavior of v when $\omega = 0$ and $\xi \neq 0$ or when $\xi \cdot \omega \neq 0$: it decays faster outside a infinite paraboloidal region behind the body, representative of the wake.

Besides (P1) and (P2), other physically relevant problems, where information about the decay structure of v and p is important, are challenging from the mathematical point of view. We will briefly mention the existence of flows with finite kinetic energy, the stability and attainability of steady motions, and flow control in exterior domains.

This talk is based on joint works with G. P. Galdi (Univ. Pittburgh, USA), T. Takahashi (Inria Nancy Grand-Est, France) and T. Hishida (Nagoya University, Japan). The references below are only a small part of many contributions, by many authors, published on the last decade, about the Navier-Stokes equations around moving obstacles. In these works, the emphasis is on the effect of the rotation of the solid.

Keywords: Navier-Stokes equations, exterior domain, rotating body, generalized Oseen fundamental solution, Landau solution, asymptotic behavior

MSC 2010: 35Q30, 35Q35, 76D05, 76D07

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From Logic to Computers. From Programs to Logic*

Salvador Lucas[1]

Logic is in the origin of computer science. Hilbert's quest for "effective methods" to mechanically prove mathematical theorems stimulated some of the brightest minds of the 20th century to find a solution or proving it unfeasible. The second option won.

Eigthy years ago, Church and Turing advanced the *thesis* that lambda calculus and a-machines (i.e., Turing Machines) formalize the fuzzy notion of "effective method" [2, 6]. They displayed logical sentences which could not be proved or disproved by any automatic means. This was the end of Hilbert's dream. And the beginning of *Computer Science*.

The Universal Turing Machine [6] was then conceived as the abstract model of our current notion of computer. Ten years later (1946) the first realization was proposed by von Neumann and his collaborators [1]. Twenty years later (1956) the first user's manual of a programming language (FORTRAN) was delivered. Thirty years later (1966) Peter Naur was calling for an increasing role of mathematical reasoning in program development [5]. He said: "We cannot indefinitely continue to build on sand." Actually, this call was followed by many others in a new quest for logical-based methods to prove program correctness and improve reliability of software [3, 4].

The main purpose of this talk is making explicit the interplay between logic and programming at different levels: program design, program semantics, program verification, and the development of tools to build reliable software systems.

Keywords: Automation, Logic, Program Analysis, Program Verification

MSC 2010: 03B35, 03B70, 68N30, 68Q05, 68Q60

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The Challenge of Developing Applicable Parametrization Algorithms for Algebraic Curves and Surfaces*

J. Rafael Sendra[1]

From a mathematical point of view, many of the problems concerning computer aided design and computer aided modeling (CAD/CAM) systems are related to the manipulation of geometric objects, mainly curves, surfaces and their combinations, in two or three-dimensional space. In particular, the interest is focused on parametric curves and surfaces since they admit two different representations (namely, parametric and implicit) and depending on the type of problem to be solved one representation has better advantages than the other (see [1, 2, 7]). This phenomenon has motivated a reciprocal relationship of interest between the fields of applications and development of constructive methods in algebraic geometry.

As a consequence, the development of, both symbolic and/or hybrid symbolic-numeric, algorithms for dealing with curves and surfaces has turned to be an active research area. Within this theoretical framework one may find algorithms: for deciding the rationality by means the genus (for curves) and the arithmetic genus and the plurigenus (for surfaces) (see [5, 6]); for computing rational parametrizations (see [5, 6]), for particular computer aided geometric design constructions (see [1, 2]), as well as for reparametrizating parametrizations under different optimality criteria (see [5]). In addition, radical parametrizations are also being studied, as well as approximate parametrizations for almost zero genus curves (see [3, 4]).

In this talk we plan to give a panoramic overview of the state of the art of the field. We will describe the main ideas of some of the current contributions, and we will try to motivate and describe the new challenges in the field.

Keywords: Algebraic Curve, Algebraic Surface, Rational Parametrization, Symbolic Algorithm, Geometric Design

MSC 2010: 68W30, 14Q05, 14Q10

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Variational methods for non-variational problems

Pablo Pedregal[1]

Starting from a classical situation of a regular functional giving rise to an underlying (variational) PDE problem, we pretend to treat the more general situation of a non-variational problem (no natural underlying energy) through typical methods tailored for variational problems. This amounts to introducing an artificial functional whose main feature is to measure how far admissible functions are from being a solution of the non-variational PDE. We will then apply typical variational techniques to this error functional to recover results for the non-variational problem, similar to the classical ones for its variational counterpart.

Keywords: Variational problem, weak solution, strong convergence

MSC 2010: 35J60, 35A15

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Algoritmos em complexos simpliciais finitos

Pedro V. Silva[1]

Os complexos simpliciais finitos são muito mais complicados do que parecem: muitos problemas naturais são insolúveis, como por exemplo decidir se um complexo (de dimensão 2) é simplesmente conexo. Introduziremos a classe dos complexos simpliciais com representação booleana, que contém os matróides (finitos) como caso particular. Esta classe goza de interessantes propriedades combinatórias, geométricas, topológicas e algorítmicas, o que constitui uma combinação rara.

Keywords: Complexo simplicial, matróide, representação booleana, grupo fundamental, shellability, tipo de homotopia

MSC 2010: 05B35, 05E45, 14F35, 15B34, 55P15, 55U10, 57M05

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S1
Algebra and Combinatorics

Algebra is one the formal pillars of mathematics, and is very ancient. Modern algebra has its roots in the mathematics of the ancient world, arising out of the basic problem of solving equations. It is now a vibrant, multi-faceted and wide-ranging branch of mathematics, having ties with almost every field of mathematics and computer science.

Fundamentally the purpose of algebra always was to produce a formula that one could put into a machine, turn a handle and get the answer. The main interest of the algebra are algebraic number theory - study of algebraic structures to algebraic integers, algebraic topology – study of topological spaces, algebraic geometry – study of algebra and geometry combined.

The entire modern world relies on combinatorial algorithms. Combinatorics turns out to be pretty relevant for group theory, probability, graph theory, topology, analysis, etc. Another important application of combinatorics is in representation theory, symmetric functions, and the study of varieties with lots of symmetries. A particular strength of the Algebra and Combinatorics is the computational and algorithmic aspects of groups and graphs.

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Hochschild cohomology of rings of differential operators in one variable

Samuel A. Lopes[1]

A polynomial h in the variable x determines the derivation $h\frac{d}{dx}$ of the polynomial ring $\mathbb{F}[x]$ and, together with the multiplication by x operator, it generates a noncommutative algebra A_h whose elements can be written as differential operators with coefficients in $\mathbb{F}[x]$. I will talk about some features of this algebra related to derivations and the structure of the Hochschild cohomology Lie algebra of A_h, both in prime and zero characteristics. I will then explain how the complete Hochschild cohomology can be determined and what this says about the formal deformations of A_h.

This is joint work with G. Benkart and M. Ondrus, and work in progress with A. Solotar.

Keywords: Hochschild cohomology, Weyl algebra, Witt algebra, derivations, Ore extensions

MSC 2010: 16S32, 16W25

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Ill-posed points for the Rational Interpolation Problem

Teresa Cortadellas Benítez[1], **Carlos D'Andrea**[2], **Eulàlia Montoro**[2]

For pairs of points $(u_1, v_1), \ldots, (u_n, v_n)$ with coordinates in a field K, with $u_i \neq u_j$, and $1 \leq k \leq n$, if $i \neq j$, the Rational Interpolation Problem consists in deciding whether there exist (and if so, compute) polynomials $N_{k-1}(x)$, $D_{n-k}(x) \in K[x]$ of degree bounded by $k-1$ and $n-k$ respectively such that $D_{n-k}(u_i) \neq 0$ for all $i = 1, \ldots, n$ and

$$\frac{N_{k-1}(u_i)}{D_{n-k}(u_i)} = v_i,\ i = 1, \ldots, n. \tag{1}$$

When $k = n$, this problem reduces to the well-known *Lagrange Interpolation* problem. In contrast with this classical problem, there is not always a solution for the Rational Interpolation Problem for any given input data. In this talk, we will present the problem, show some algebraic formulations of it, and a geometric description of the set of ill-posed points (those for which there is no solution to (1)).

Keywords: Rational Interpolation, Vandermonde Matrices, Weak Rational Interpolation, Minimal Solutions, Filtrations

MSC 2010: 13P15, 15A15, 68W30

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Combinatorial and computational properties of the sylvester monoid

Alan J. Cain[1], **Robert Gray**[2], **António Malheiro**[1]

The sylvester monoid is the monoid of right strict binary search trees. It was introduced to give a new construction of the Loday–Ronco hopf algebra, and is one of a family of 'plactic-like' monoids whose elements can be identified with combinatorial objects [2].

This talk will discuss recent work on the sylvester monoid from two perspectives: computing and combinatorics. On the computational side, it will discuss how the sylvester monoid is automatic (in the sense of the automatic groups of Epstein et al. [1]) and is presented by a convergent rewriting system. On the combinatorial side, it will discuss the 'cyclic shift' relation, which relates elements that factor as xy and as yx. If one builds a graph whose vertices are the elements of the sylvester monoid, and whose edges are given by the cyclic shift relation, then the connected components of this graph are finite and have a very neat characterization, and the connected components have bounded diameter (although the number of vertices in a component is unbounded).

Keywords: sylvester monoid, binary search trees, automatic, cyclic shifts

MSC 2010: 20M35, 05C12

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The number of parking functions with center of a given length

Rui Duarte[1], **António Guedes de Oliveira**[2]

Let $1 \leq r \leq n$ and suppose that, when the *Depth-first Search Algorithm* is applied to a given rooted labelled tree on $n+1$ vertices, exactly r vertices are visited before backtracking. Let R be the set of trees with this property. We count the number of elements of R.

For this purpose, we first consider a bijection [3], due to Perkinson, Yang, and Yu, that maps R onto the set of parking function with *center* [1] of size r. A second bijection maps this set onto the set of parking functions with *run* r. We then prove that the number of length n parking functions with a given run is the number of length n *rook words* [2] with the same run. This is done by counting related lattice paths in a ladder-shaped region. We finally count the number of length n *rook words* with run r, which is the answer to our initial question.

Keywords: parking functions, bijections

MSC 2010: 05A15, 05A19, 05C30

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Spherical functions and Latin squares

Carlos Gamas[1]

The Alon-Tarsi Conjecture states that for even n, the number of even latin squares of order n differs from the number of odd latin squares. In this note we prove that this conjecture is true if and only if there exists a permutation $\zeta \in S_{n^n}$ and a spherical function, φ, such that $\varphi(\zeta) \neq 0$.

Keywords: spherical functions, latin squares

MSC 2010:

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The non-abelian tensor product of different structures

Xabier García-Martínez[1]

The non-abelian tensor product of Lie algebras was introduced by Ellis in [1]. In this talk we are going to introduce the different generalizations of this object to several structures as Lie superalgebras, Leibniz algebras and superalgebras and Lie-Rinehart algebras. The non-abelian tensor product is a very important tool that relates the Lie algebra homology with cyclic homology, non-abelian homology or central extensions. We are going to use this object to have simpler proofs of known theorems as well to obtain new results.

Keywords: Non-abelian tensor and exterior products, Lie superalgebras, homology

MSC 2010: 17B55, 17B30, 17B60

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6th Iberian Mathematical Meeting
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On the classification of nilpotent quadratic Lie algebras

Daniel de-la-Concepción[1], **Pilar Benito**[1], **Jesús Laliena**[1]

A key fact about simple (semi-simple) Lie algebras is that an invariant non-degenerate bilinear form, the Killing form, can be defined on them. The existence of such a form is a main ingredient for the physical theories associated to them.

Nevertheless, there are more Lie algebras with such a nice bilinear form. Such a structure is called a quadratic Lie algebra and it is a pair (L, B) where L is a Lie algebra and $B : L \times L \to L$ is a non-degenerate bilinear form invariant in the sense that $\forall x, y, z \in L,\ B([x, y], z) = B(x, [y, z])$.

In this talk I will concentrate in the nilpotent Lie algebras and I will show the complexity of classifying such algebras by stating an equivalence of categories that relates them to orbits of the group of automorphism of the free Lie algebra acting on a specific Lie module.

Keywords: Quadratic Lie algebra, Invariant Bilinear form, Equivalence of categories

MSC 2010: 17B30, 15A63

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On the growth of the Kronecker coefficients

Mercedes Helena Rosas[1]

We will introduce the Kronecker coefficients, and explore some of their occurrences in the theory of symmetric functions, and in the representation theory of the general lineal group, and the symmetric group,. Then we will present some of their main known properties. Finally we will report on some recent results on the rate of growth experienced by the Kronecker coefficients as we add cells to the rows and columns indexing partitions. This is join work with Emmanuel Briand and Amarpreet Rattan.

Keywords: Representations theory of the symmetric group, symmetric functions

MSC 2010: 05E10, 05E05

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Combinatorics, Number Theory and Groups: the other Name of Semigroups

João Araújo[1]

During the XIXth and early XXth centuries some remarkable breakthroughs came from the idea, probably first considered by Galois, of extracting information about an object from its *morphisms*. Therefore it came as no surprise that a conviction started spreading: *to every mathematical object there is some semigroup attached, and in many cases that semigroup contains very important information about the original object.* This rough idea was reduced to a mathematical question by Stanislaw Ulam [5] when he proposed the general problem of checking when a natural class of mathematical objects has the property that

$$\mathrm{End}(A) \cong \mathrm{End}(B) \Rightarrow A \cong B\,.$$

In words,*which natural classes of mathematical objects possess the property that the endomorphism monoid of an object contains enough information about the object itself to the point of uniquely identifying it?*

However, this expressive power of semigroups was viewed as being more of a weakness than that of a strength. If nearby every mathematical object is a semigroup encoding critical information about the object itself, then semigroups encode critical information about the whole of mathematics and hence non-trivial statements about them compare to non-trivial statements about *all* of mathematics.

After some preparatory results obtained both in the East and in the West, this depressing view of semigroups was dramatically changed by J.A. Green and D. Rees. Not only important and deep results could be proved about semigroups, but —the good news— their local structure is quite transparent and well described. In the words of John Rhodes *we fully understand the local structure of semigroups*. Therefore the depressing meta-mathematical considerations were replaced by an equally meta-mathematical euphoria:

- nearby every object is a semigroup encoding relevant information about the object;
- many mathematical objects are very important to the world (computer science, physics, etc.);
- therefore semigroups are very important to the world and, despite what was previously thought, they are tractable so that to help the world we have to dive into semigroup theory.

As a consequence, for more than three decades papers in semigroup theory consistently appeared in the most reputable mathematics journals. The background of Green (group theory) and of Rees (ring theory) to a large extent shaped the future work on semigroups; in the words of John Rhodes, semigroups started to be seen as *defective groups* or as *the multiplicative part of a ring*.

To study an object (for example, the natural numbers) we start by identifying a relevant subobject (set of prime numbers) that in some sense controls the behaviour of the whole object. In semigroup theory there are two obvious candidates to be taken as this relevant subset: the group of units (the non-defective part of the defective group) or the idempotents. *Prima facie* all bets should be put on the group of units. It is well known that semigroups are oriented, with parts *above* and parts *below*; it is also known that the parts *below* never generate the parts *above* (rank 1 matrices do not generate rank 2 matrices); the group of units is above everything else; a semigroup S with group of units G is in fact the union $S = \cup_{a \in S \setminus G} \langle \{a\} \cup G \rangle$; and groups are among the most studied of all classes of algebras. On the other hand, idempotents, in general, do not even form an algebra (the product of two idempotent matrices does not need to be idempotent), and they were even less well understood than groups, then... and now! There was only one small detail left: the nature of the questions semigroups pose to groups.

As a sample consider the following ones (where $N := \{1, \ldots, n\}$, $\mathcal{S}_n$ denotes the symmetric group on N, and $\mathcal{T}_n$ denotes the full transformation semigroup on N):

1. Let $i \leq j \leq n$. Classify the groups $G \leqslant \mathcal{S}_n$ such that for every i-set $I \subseteq N$ and for every j-set $J \subseteq N$, there exists $g \in G$ such that $Ig \subseteq J$; of course, when $i = j$ these are just the i-homogeneous groups.

2. Let $k < n$ be two natural numbers. Classify the groups $G \leqslant \mathcal{S}_n$ such that in the orbit of every k-set contained in N there exists a section for every k-partition of N; when $k = 2$ this property is just primitivity; but what happens with the remaining k?

3. Let λ be a partition of n. Classify all the groups $G \leqslant \mathcal{S}_n$ transitive on the type λ partitions of N.

4. Classify the primitive groups $G \leqslant \mathcal{S}_n$ such that there exists a partition P of N and a set $S \subseteq N$ such that Sg is a section for P, for all $g \in G$.

5. Classify the primitive groups with diameter at most n and that do not satisfy the previous property. (The diameter of a finite group is the maximum diameter of its Cayley graphs.)

6. Consider a primitive group $G \leqslant \mathcal{S}_n$ and a transformation $a \in \mathcal{T}_n \setminus \mathcal{S}_n$ such that the kernel type of a is non-uniform. Is it true that $\langle a, G \rangle$ generates a constant map?

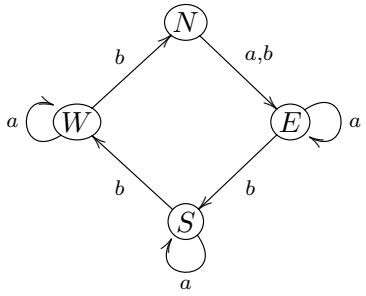
7. Classify the groups $G \leqslant \mathcal{S}_n$ such that for all non-invertible transformations $a \in \mathcal{T}_n \setminus \mathcal{S}_n$ we have

$$\langle \{a\} \cup G \rangle \setminus G = \langle g^{-1}ag \mid a \in G \rangle.$$

Now, for a moment, let's go back to the sixties or early seventies and imagine some semigroupist asks you any of these questions; would you find them anything but a brick wall? Apparently that was what happened and hence semigroupists turned to the study of how idempotents shape the structure of a semigroup. John M. Howie's book can be seen as a brilliant survey of that approach.

The fact that these questions are very challenging does not imply *per se* that these are relevant problems. But maybe they are.

It is well known that ordered pears are sold at an higher price than unordered pears. Therefore, to help farmers moving up the value chain we have to provide them with a robust device that receives pears in any order and delivers all of them oriented. Here, just for the sake of explanation, suppose that when the pears enter in a conveyor belt each one is either pointing to the North, or to the South, or to the West or to the East; of course farmers deal with a much more complex situation, but mathematicians have yet to reach their level. Now it is possible to order the pears just using obstacles of two different kinds along the pears' trip on the conveyor belt, modelled by the following automaton (a step-by-step construction and explanation of this device can be found in [1], pages 24–26):



Observe that if you follow the sequence ab^3ab^3a, you will always end in E. This means that if all the pears are subjected to that sequence of obstacles, they all will end up pointing to the East, irrespective of their original orientation. Such a sequence is called a *reset word*.

One of the oldest conjectures in automata theory, the Černý conjecture (published in 1964), says that a given automaton on n states either does not admit a reset word, or there exists one of length $(n-1)^2$. Observe that the automaton above has 4 states and the given reset word has length $(4-1)^2$; in addition it can be seen that there is none shorter; therefore, the bound $(n-1)^2$, if true, would be optimal. An enormous effort has been made to prove this conjecture, but so far without success. One step towards the solution would be to solve problem (5) above.

However, suppose the Černý conjecture turned out to be true and a farmer approaches your department with a real world (non-simplified) automaton with millions of states and asks if it admits a reset word. It would be extremely vexing if the department replies: *We do not know if that automaton admits a reset word; but if so, then its length is at most* $(n-1)^2$. Therefore, to avoid sounding funny, in addition to (5) mathematicians really need to answer questions (4) and (6) above (please see [2, 3, 4]). However, as previously observed, in 1964 these questions would have looked like a solid brick wall.

The first step towards a change of the tide occurred in 2005; I went to a conference in St. Andrews and in an idle conversation, a group theorist told me that *with the classification of finite simple groups, we can answer any question on finite permutation groups*. Great! Such a bold statement, that probably he would not repeat today, was instrumental for me to unearth some of the questions above: if now they can answer everything, the wall will fall down.

The second step occurred in 2006 in the banquet in honor of a marvelous supervisor, John Fountain. By accident I sat next to Stephen Donkin and after introducing ourselves, I commented *I am glad you are a group theorist because I have a number of questions in group theory.* He just replied *try me!* and I shot question (4) above. He thought for a moment and then said: *that is a very interesting question; you should ask it to Peter Neumann in Oxford because he is going to be interested.* And in the end of the meal he reinforced: *please do not forget to write to Peter Neumann; if you have difficulties please write to me.* But there were no difficulties: Peter Neumann replied almost instantaneously! The wall was severely shaken [4].

In this talk I will give a brief survey of what has been going on in this part of mathematics.

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6th Iberian Mathematical Meeting
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Jordan techniques in Lie theory

Antonio Fernández López[1]

There are no Jordan algebras; there are only Lie algebras (I. L. Kantor). Of course this can be turned around; *nine times out of ten, when you open up a Lie algebra you find a Jordan algebra inside which makes it tick* (K. McCrimmon).

As will be seen in this talk, the dictum of McCrimmon is quite correct. We associate to any ad-nilpotent element of index 3 of a Lie algebra, a Jordan algebra [3] that inherits all good properties of the Lie algebra and reflects the nature of the Jordan element in its structure. Since this transference of properties also works in the opposite direction, we have at our disposal a useful device to deal with Lie problems by means of Jordan theory.

Due to the short duration of the talk, I will only present, as an application of this technique, a proof [2] of the existence of extremal elements in finitary Lie algebras [1]. I will also mention a recent paper of E. Zelmanov [4] that significantly extends the positive solution of the Restricted Burnside Problem, and where the Jordan-Lie connection plays a fundament role.

Keywords: Lie algebras, Jordan algebras, finitary algebras

MSC 2010: 17B65, 17C10

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Reduction theorems on generalised kernels of finite semigroups

Vicente Pérez Calabuig[1]

The problem of computing kernels of finite semigroups goes back to the early seventies and became popular among semigroup theorists through the Rhodes Type II conjecture which proposed an algorithm to compute the kernel of a finite semigroup with respect to the class of all finite groups. Proofs of this conjecture were given in independent and deep works by Ash and Ribes and Zalesskiĭ, and the results of these authors that led to its proof have been extended in several directions. A general treatment of the question is presented for any variety of groups as well as reduction theorems that reduce the problem to simpler structures.

This is joint work with professor Adolfo Ballester-Bolinches.

Keywords: Finite semigroup, Kernels, Inverse semigroup, Varieties

MSC 2010: 20M07, 20M10, 20M17, 20M18

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Evolution algebras of arbitrary dimension and their decompositions

Yolanda Cabrera Casado[1]

We study evolution algebras of arbitrary dimension. We analyze in deep the notions of evolution subalgebras, ideals and non-degeneracy and describe the ideals generated by one element and characterize the simple evolution algebras. We also prove the existence and unicity of a direct sum decomposition into irreducible components for every non-degenerate evolution algebra . When the algebra is degenerate, the uniqueness cannot be assured.

The graph associated to an evolution algebra (relative to a natural basis) will play a fundamental role to describe the structure of the algebra. Concretely, a non-degenerate evolution algebra is irreducible if and only if the graph is connected. Moreover, when the evolution algebra is finite-dimensional, we give a process (called the fragmentation process) to decompose the algebra into irreducible components.

Keywords: Evolution algebra, evolution subalgebra, evolution ideal, non-degenerate evolution algebra, simple evolution algebra, graph associated, reducible evolution algebra, irreducible evolution algebra

MSC 2010: 17D92

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Automated proof and discovery in dynamic geometry*

Francisco Botana[1]

The talk describes the application of algebraic approaches to a geometric interface with millions of users worldwide. More specifically, new abilities concerning automatic proof and discovery in GeoGebra are detailed. A taxonomy for the exact computation of plane algebraic loci and a protocol for computing plane envelopes are recalled. Several methods currently implemented in GeoGebra for general proving are discussed, and our first results dealing with automatic discovery in elementary diagrams are also described.

This is joint work with several colleagues of our MTM2014-54141-P research group and with colleagues from the GeoGebra developers team.

A few, closely related, references are appended in the bibliography.

Keywords: Automatic reasoning in geometry, Interactive learning environments, Dynamic geometry

MSC 2010: 68W30, 68T35

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Combinatorial computer algebra for network analysis and percolation

Fatemeh Mohammaddi[1], **Eduardo Sáenz-de-Cabezón**[2], **Henry Wynn**[3]

The study of networks and other coherent systems using tools from combinatorial commutative algebra has proven to be a fruitful area of research in the last years. The main idea is to associate a monomial ideal to the system under study and obtain its properties (reliability, components' importance, robustness, etc) by studying the algebraic features of the ideal (Hilbert series and function, resolutions, primary decompositions, etc). When applying the results of this approach to medium and large size problems one needs a computer algebra approach. In this respect, the algorithms of commutative computer algebra appear to be efficient for the problems treated.

Keywords: Monomial ideals, Hilbert series, computer algebra, networks, algebraic reliability, percolation

MSC 2010: 13P25, 13D02, 05E40, 60K35

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Gray codes for noncrossing and nonnesting partitions of classical types

Alessandro Conflitti[1], **Ricardo Mamede**[2]

A Gray code is a listing structure for a set of combinatorial objects such that some consistent (usually minimal) change property is maintained throughout adjacent elements in the list. I shall present combinatorial Gray codes and explicit designs of efficient algorithms for lexicographical combinatorial generation of the sets of noncrossing and nonnesting set partitions of length n and types A, B and D. This is a joint work with Alessandro Conflitti.

Keywords: Gray code, Hamilton cycle, Weyl groups, noncrossing partition, nonnesting partition.

MSC 2010: 05A18, 68W99, 94B25.

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Universal α-central extensions of Hom-Leibniz n-algebras*

N. Pacheco Rego[1], **J. M. Casas**[2]

Deformations of algebra structures by means of endomorphisms give rise to Hom-algebra structures. They are motivated by discrete and deformed vector fields and differential calculus. Part of the reason to study Hom-algebras is its relation with the q-deformations of the Witt and the Virasoro algebras (see [9]). In this way, deformations of algebras of Lie and Leibniz type were considered, among others, in [5, 9]. The generalizations of n-ary algebra structures, such as Hom-Leibniz n-algebras (or n-ary Hom-Nambu) have been introduced in [1] as triples $(\mathcal{L}, [-,\dots,-], \widetilde{\alpha})$ consisting of a $\mathbb{K}$-vector space $\mathcal{L}$ equipped with an n-linear map $[-,\dots,-] : \mathcal{L}^{\times n} \longrightarrow \mathcal{L}$ and a family $\widetilde{\alpha} = (\alpha_i), 1 \leq i \leq n-1$ of linear maps $\alpha_i : \mathcal{L} \longrightarrow \mathcal{L}$, satisfying the following fundamental identity:

$$\begin{gathered}[[x_1, x_2, \dots, x_n], \alpha_1(y_1), \alpha_2(y_2), \dots, \alpha_{n-1}(y_{n-1})] = \\ \sum_{i=1}^{n} [\alpha_1(x_1), \dots, \alpha_{i-1}(x_{i-1}), [x_i, y_1, y_2, , \dots, y_{n-1}], \alpha_i(x_{i+1}), \dots, \alpha_{n-1}(x_n)]\end{gathered} \tag{1}$$

for all $(x_1, \dots, x_n) \in \mathcal{L}^{\times n}, y = (y_1, \dots, y_{n-1}) \in \mathcal{L}^{\times(n-1)}$.

When these twisting maps are all equal to the identity map, then one recovers Leibniz n-algebras [7]. In case $n = 2$, identity (1) is the Hom-Leibniz identity (2.1) in [5], so Hom-Leibniz 2-algebras are exactly Hom-Leibniz algebras.

The goal of this talk is to introduce and characterize universal α-central extensions of Hom-Leibniz n-algebras. In case $n = 2$ we recover the corresponding results on universal α-central extensions of Hom-Leibniz algebras in [5]. Moreover, in case $\alpha = Id$ we recover results on universal central extensions of Leibniz n-algebras in [3] and, in case $n = 2$ and $\alpha = Id$, we recover results on Leibniz algebras from [4].

The talk is organized as follows: we introduce the necessary basic concepts on Hom-Leibniz n-algebras and construct the homology with trivial coefficients of Hom-Leibniz n-algebras. Bearing in mind [2], we endow a Hom-Leibniz n-aslgebra $\mathcal{L}$ with a structure of $(\mathcal{D}_{n-1}(\mathcal{L}) = \mathcal{L}^{\otimes n-1}, \alpha')$-symmetric Hom-co-representation as Hom-Leibniz algebras and define the homology with trivial coefficients of $\mathcal{L}$ as the Hom-Leibniz homology $HL^{\alpha}_{*}(\mathcal{D}_{n-1}(\mathcal{L}), \mathcal{L})$.

Based on the investigation initiated on [5] and motivated by the fact that the category of Hom-Leibniz n-algebras doesn't satisfy the so called in [8] **UCE** condition, namely the composition of central extensions is central, we generalize the concepts

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of (α)-central extension, universal (α)-central extension and perfection to the framework of Hom-Leibniz n-algebras. We also extend the corresponding characterizations of universal (α)-central extensions. In particular, we show their interplay with the zeroth and first homology with trivial coefficients.

We also introduce the concept of non-abelian tensor product of Hom-Leibniz n-algebras that generalizes the non-abelian tensor product of Hom-Leibniz algebras in [6] and Leibniz n-algebras [3], and we establish its relationship with the universal central extension.

Keywords: Hom-Leibniz n-algebra, universal (α)-central extension, non-abelian tensor product, unicentral Hom-Leibniz n-algebra

MSC 2010: 17A30, 17B55, 18G60

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6th Iberian Mathematical Meeting
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Regular pseudo-oriented maps

António Breda d'Azevedo[1]

Despite not being a topological property, pseudo-orientability (introduced by Steve Wilson in the eighties to distinguish some nonorientable maps) has interesting resemblances with orientability when restricted to maps (cellular embeddings of "multiple" graphs on compact connected surfaces). The classification of orientable maps is in a very advanced stage, however this does not happen to pseudo-orientable maps. In this talk I speak about the classification of regular pseudo-oriented maps of small genus and of prime characteristic.

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Combinatorial conditions for linear systems of projective hypersurfaces

Miguel A. Marco-Buzunariz[1]

It is well known that five points in generic position in a projective plane determine one conic. Four points determine an infinite family of such conics, called a pencil. The pencils of curves have been a subject of study since the very beginning of algebraic geometry.

One of the usual problems related with their study is the determinacy of conditions for three or more curves to belong in a pencil. The first important result in this direction was given by M. Noether ([4]) in his famous "$AF + BG$" theorem, that reduced this problem to check local conditions on the base points of the pencil.

In the current century, there have been several improvements to this result that weakened this local conditions to purely combinatorial ones. First for the specific case of line arrangements ([2]) and later for general curves ([1]).

The talk will survey these results and introduce their possible generalizations to higher dimensions; including some known results and open conjectures.

Keywords: Projective varieties, Pencils

MSC 2010: 14N10

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On ω-identities over finite aperiodic semigroups with commuting idempotents

José Carlos Costa[1], **Mário J. J. Branco**[2]

The notion of *pseudovariety* plays a key role in the classification of finite semigroups. Recall that a pseudovariety of semigroups is a class of finite semigroups closed under taking homomorphic images of subsemigroups and finitary direct products. An inverse semigroup is a regular semigroup whose idempotents commute. In [1], Ash proved that the pseudovariety generated by finite inverse semigroups is precisely the pseudovariety $\mathbf{ECom}$ of idempotent commuting semigroups. Let $\mathbf{AInv}$ be the pseudovariety generated by finite aperiodic (i.e., group free) inverse semigroups and let $\mathbf{AECom}$ be the pseudovariety of aperiodic semigroups with commuting idempotents. Surprisingly, Higgins and Margolis [2] have shown that these pseudovarieties are not equal. They did it by exhibiting a certain finite aperiodic semigroup with commuting idempotents and by showing that it does not divide any finite aperiodic inverse semigroup.

In this talk we will present a new proof of the above result by showing that the pseudovarieties $\mathbf{AInv}$ and $\mathbf{AECom}$ do not satisfy the same pseudoidentities. For this we study the word problems for ω-terms (formal expressions obtained from letters of an alphabet using only the operations of multiplication and ω-power) over each of these pseudovarieties. It is shown that both problems are decidable and have different solutions.

MSC 2010: 20M05, 20M07, 68Q70

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S2

Applied Partial Differential Equations in Fluids and Materials

Partial differential equations are a essential tool in the modelling of phenomena in nature and industrial processes. An important part of these applications corresponds to fluids and materials. Both fields, equations and applications, need to be connected in order to advance in knowledge. The session brings together different aspects, theory, numerics, and applications to fluids and materials with this purpose.

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Effects of ambient rotation and temperature gradients on the development of a single or double-celled vortex and multiple vortices

D. Castaño[1], **M. C. Navarro**[2], **H. Herrero**[3]

The generation of single or double-celled vortices and multiple vortices in a cylinder non-homogeneously heated from below in a rotation frame is shown in this talk. In a moderate rotation frame, the increase of thermal gradients is the responsible for the progression from a one-celled vortex, characterized by an updraft at the centerline, to a two-celled vortex, characterized by a central eye. If the thermal gradients continue increasing, the axisymmetric eyed-vortex loses the axisymmetry, the eye displaces from the center and tilts. For larger rotation rates, the axisymmetric single-celled vortex does not progress to a double-celled vortex when the thermal gradients increase, but it becomes into a double vortex. The change in the vortex type is also observed in laboratory tornadoes. In [1] it is shown the progression, as the swirl ratio increases, of vortex structure from a single cell to a two-cell vortex with a downdraft in the center. Our findings are relevant for their connection with the formation and evolution of some atmospheric vortices. The numerical results have been obtained by direct numerical simulation of the time-dependent Navier-Stokes equations coupled with the heat equation, and the boundary conditions. The equations have been solved using a second-order time-splitting method proposed in [2], and described and tested in [3] for a cylindrical configuration. For the spatial discretization a pseudo-spectral method is used, with a Fourier expansion in the azimuthal coordinate ϕ and Chebyshev collocation in r and z.

Keywords: Vortices, Thermoconvective instabilities, Navier-Stokes equations

MSC 2010: 74S25, 76B47

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Thermodynamical behaviour of polythermal ice sheets by using temperature and enthalpy formulations

N. Calvo[1], **J. Durany**[1], **C. Vázquez**[2]

In this work an original enthalpy formulation for the thermal behaviour of polythermal ice sheets is proposed and numerically solved. Although the modelling follows some ideas introduced in [1], nonlinear basal boundary conditions in both cold and temperate regions are also considered, thus including the sliding effects in the frame of a fully coupled shallow ice approximation (SIA) model. Then, the highly nonlinear system of partial differential equations governs three main thermomechanical problems: the upper profile evolution, the ice velocity field and the enthalpy distribution. One of the main novelties of this work comes from the introduction of the Heaviside multivalued operator to take into account the discontinuity of the thermal diffusion function at the cold-temperate transition surface (CTS) free boundary. Moreover, we propose a duality method for maximal monotone operators to solve simultaneously the nonlinear diffusive term and the free boundary. In addition, spatial finite element discretizations and Newton' s method for solving the nonlinear system at each time step are performed. Some numerical simulation examples with real data from Antarctica are presented and illustrate the small differences between the computed results from the enthalpy formulation and the alternative formulation in terms of the temperature [2].

Keywords: Ice sheet models, free boundaries, enthalpy, finite elements, duality methods

MSC 2010: 86A40, 35R35, 65N60, 49M29

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Drug release from viscoelastic polymeric platforms: non-Fickian solvent absorption and Fickian drug desorption

J. A. Ferreira[1], **M. Grassi**[2], **P. de Oliveira**[3], **G. Romanazzi**[1,4]

We consider a polymeric spherical platform containing a solid dispersed drug that is in contact with a solvent fluid. The solid drug in contact with the solvent fluid dissolves and the Fickian release of dissolved drug takes place. While swelling, a non Fickian sorption of the solvent molecules occurs induced by the effect of the viscoelastic properties of the polymer. The fluid entrance, the drug dissolution and the drug release to an external environment are described by a system of PDE's complemented with an equation for the swelling front, initial and boundary conditions. The model includes the two major factors that govern a swelling process of a polymeric platform within a release medium: the cross link density and the concentration of the external medium. Energy estimates for the mass of solvent fluid, undissolved and dissolved drug in the polymeric platform are established. Numerical simulations that illustrate our theoretical results are also included.

Keywords: Drug release, viscoelastic polymers, non-Fickian diffusion, Fickian diffusion

MSC 2010: 35K55, 65N06

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Evolution of secondary whirls in dust devil-like vortices in a route to chaos

D. Castaño[1], **M. C. Navarro**[1], **H. Herrero**[1]

The appearance, evolution and disappearance of periodic and quasiperiodic dynamics of fluid flows in a cylindrical annulus locally heated from below are analysed using nonlinear simulations. The results reveal a route of the transition from a steady axisymmetric vertical vortex to a chaotic flow. The chaotic flow regime is reached after a sequence of successive supercritical Hopf bifurcations to periodic, quasiperiodic and chaotic flow regimes. A scenario similar to the Ruelle-Takens-Newhouse scenario [3] is verified in this convective flow. In the transition to chaos we find the appearance of subvortices embedded in the primary axisymmetric vortex, flows where the subvortical structure strengthens and weakens, almost disappears before reforming again, leading to a more disorganized flow to a final chaotic regime. Results are remarkable as they connect to observations describing the formation, weakening and virtually disappearance before revival of subvortices in some atmospheric swirls such as dust devils [4]. The numerical results have been obtained by direct simulation of the timedependent governing equations, incompressible Navier-Stokes coupled with a heat equation under Boussinesq approximation. These equations have been solved using the second-order time-splitting method. A pseudo-spectral method is used for the spatial discretization, with a Fourier expansion in the azimuthal coordinate and Chebyshev collocation in the radial and vertical coordinates. These works have been published in [1, 2].

Keywords: natural convection, spectral methods, vertical vortex

MSC 2010: 74S25, 76B47

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An IMEX-WENO scheme for the equilibrium dispersive model of chromatography

F. Guerrero[1], **P. Mulet**[1], **R. Donat**[1]

The nonlinear equilibrium dispersive model consists of a partial derivative equation in the form of viscous conservation law. The solutions of this type of PDEs may contain very sharp transitions and this is the case in this chromatographic model. We show that a high resolution Weighted Essentially Non Oscillatory (WENO) scheme gives accurate numerical solutions which capture the sharp discontinuities present in the chromatographic fronts. Moreover, since the dispersive term in the model imposes a severe restriction in the time step size, we apply an implicit strategy to handle the parabolic term and an explicit one for the convective term to improve the efficiency of the numerical scheme. In addition, we show a series of numerical experiments to test different aspects of the solutions in order to ensure the correct behavior of our IMEX-RK2 scheme.

Keywords: Conservation laws, WENO schemes, IMEX schemes, Numerical schemes, Chromatography

MSC 2010: 35L40, 35L45, 35L65, 35L67, 65M06, 65M22, 65F10, 65H17

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Stability and bifurcation of a capillary fluid on an infinite cylinder

Rafael López[1]

We consider a fluid of an incompressible and homogeneous liquid partially wetting the exterior surface on an infinite solid cylinder Σ where the only force is the surface tension and the effect of gravity is negligible. The shape of the air-liquid interface M arises from minimizing the energy $E = |M| - c|W|$, where $|M|$ is the area of M, $|W|$ is the area of the wetted region by the fluid on Σ, and the constant $c \in (-1, 1)$ depends on the material of Σ. The minimization is under the constraint that the volume of the fluid is fixed and the contact curves are free to move on Σ. It follows from the Young-Laplace equation that the first variation of E is zero if and only if the mean curvature H of M is constant and the angle between the unit normal vectors to M and Σ is a constant $\gamma \in (0, \pi)$ along the curve of contact, where $\cos\gamma = c$. In such a case, we say that M is a capillary surface on Σ.

In this talk we assume that the fluid is invariant in the directions of the axis of Σ. In particular, we suppose that the free surface M of the fluid is a cylinder that meets Σ (different radii) along two meridians and with contact angle γ. Among the different shapes of the interface, we are interested on stable capillary surfaces because they are the configurations that can be physically realizable. The study of the stability of a circular cylinder has appeared in many physical settings ([1, 3, 7, 8]) and it has been analyzed in [4, 5, 9]. Motivated by the classical Rayleigh instability criterion, we give a stability result in terms of the wavelength of M, determining when M is or is not stable [6]. This study is equivalent to an eigenvalue problem for the Jacobi operator with boundary conditions of Neumann type.

When the fluid loses the stability, the eigenvalues cross zero and the Jacobi equation has non-trivial solutions. In such a case, it is possible the appearance of new equilibrium shapes which bifurcate from the initial cylindrical fluid [1, 4]. Under certain configurations (radii and contact angle), we prove that the cylinder M bifurcates in a family of new surfaces of constant mean curvature and intersecting Σ with the same contact angle γ. Moreover these surfaces are periodic in the direction of the axis of Σ. The result that we use is the "bifurcation at a simple eigenvalue" due to Crandall-Rabinowitz [2]. Here the mean curvature H (or equivalently the radius r) of M is the variable in the bifurcation argument. The bifurcation appears when the second eigenvalue is 0 and, in such a case, we need to prove that the corresponding eigenspace is one-dimensional. A detailed description of the eigenvalues allows to find those configurations of the fluid where the Crandall-Rabinowitz scheme can apply.

Keywords: capillary surface, cylinder, bifurcation, stability, simple eigenvalues

MSC 2010: 58E12, 49K30, 53A10

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Analysis of a parabolic-elliptic problem with moving parabolic subdomain through a Lagrangian approach

Rafael Muñoz-Sola[1]

Let Ω be a bounded domain of $\mathbb{R}^n$ with Lipschitz boundary, $\hat{\Omega} \subset\subset \Omega$ a subdomain with smooth enough boundary, $T > 0$ and $\mathbf{X} : [0,T] \times \overline{\hat{\Omega}} \mapsto \mathbb{R}^n$ be a smooth mapping such that: $\mathbf{X}(t,\cdot)$ is injective for all $t \in [0,T]$, $\det(D_p\mathbf{X})(t,p) > 0$ for all $(t,p) \in [0,T] \times \overline{\hat{\Omega}}$, $\mathbf{X}([0,T]) \times \overline{\hat{\Omega}}) \subset \Omega$ and $\mathbf{X}(0,p) = p$ for all $p \in \overline{\hat{\Omega}}$. We denote $\Omega_t := X(t,\cdot)(\hat{\Omega})$. For $n = 3$, $\mathbf{X}$ represents a deformation of a body evolving smoothly with time. Let $\mu > 0$ be a constant, $\hat{\sigma} \in L^\infty(\hat{\Omega})$ such that $\hat{\sigma} \geq \underline{\sigma} > 0$ and let $\sigma \in L^\infty((0,T) \times \Omega)$ defined by: $\sigma(t,x) = \hat{\sigma}(p)$ if $x = \mathbf{X}(t,p)$ with $p \in \hat{\Omega}$, and $\sigma(t,x) = 0$ if $x \notin \overline{\Omega_t}$. Let $A^0 \in H^1(\hat{\Omega})$ and $f \in H^1(0,T;L^2(\Omega_S))$ be given data, where the open set $\Omega_S \subset \Omega$ and $\overline{\Omega_S} \cap \overline{\Omega_t} = \emptyset$ for all $t \in [0,T]$. The goal of this talk is to study the regularity of the following parabolic-elliptic initial-boundary value problem: find A s. t.

$$\begin{cases} \sigma \frac{\partial A}{\partial t} - \frac{1}{\mu}\triangle A = f & \text{in } (0,T) \times \Omega, \\ A = 0 & \text{on } (0,T) \times \partial\Omega, \\ A(\cdot,0) = A_0 & \text{in } \hat{\Omega}. \end{cases} \tag{1}$$

This problem is closely related to the model considered in [1] and [2] for the electromagnetic field produced by a coil in a cylindrical metallic workpiece undergoing a given deformation, under suitable assumptions of symmetry of the data. The model consists in the parabolic-elliptic PDE (written in cylindrical coordinates)

$$\sigma \frac{\partial A}{\partial t}\vec{e}_\theta + \nabla \times \left[\frac{1}{\mu}\nabla \times (A\vec{e}_\theta)\right] = J_S\vec{e}_\theta \quad \text{in } (0,T) \times (0,R) \times (0,L), \tag{2}$$

where J_S is the current density in the coil and $\vec{A} := A(t,r,z)\,\vec{e}_\theta$ is the magnetic vector potential. In [1] some results of existence, uniqueness and regularity were obtained for (2) (supplemented with an initial condition and an homogeneous Dirichlet condition), but the analysis of the convergence of the fully discrete numerical approximation introduced in [2] needs further regularity. Roughly speaking, it requires the $H^2(0,T;L^2(\Omega))$ regularity of the solution. For both problems, the fact that the PDE is parabolic only in the time-dependent set Ω_t makes difficult to study this regularity by using the approach of [1]. This approach is based in the Eulerian coordinates x for problem (1) (and in the cylindrical coordinates associated to x for problem (2)).

In this talk, we develop an approach based on the Lagrangian coordinate p to study the regularity of the solution of (1). We start from the weak formulation of (1), namely:

$$\begin{cases} \text{Find } \ A \in L^2(0;T;H_0^1(\Omega)), \text{with } \frac{\partial A}{\partial t} \in L^2(Q) \quad \text{s.t.} \\ \int_{\Omega_t} \sigma \frac{\partial A}{\partial t} v dx + \mu \int_\Omega \nabla A \cdot \nabla v dx = \int_{\Omega_S} f v dx \quad \forall v \in H_0^1(\Omega) \quad \text{a.e. } \ t \in (0,T) \\ A(0) = A_0 \quad \text{in } \hat{\Omega}, \end{cases} \tag{3}$$

where $Q = \{(t,x) \in \mathbb{R}^{n+1}, t \in (0,T), x \in \Omega_t\}$. First, since $\mathbf{X}$ is defined only in $[0,T] \times \overline{\hat{\Omega}}$, we construct an extension $\tilde{\mathbf{X}} : [0,T] \times \overline{\Omega} \mapsto \overline{\Omega}$ of $\mathbf{X}$ such that $\tilde{\mathbf{X}}$ is smooth, $\det(D_p \tilde{\mathbf{X}})(t,p) > 0$ for all $(t,p) \in [0,T] \times \overline{\Omega}$ and $\tilde{\mathbf{X}}(t,\cdot) : \overline{\Omega} \mapsto \overline{\Omega}$ is a homeomorphism keeping invariant both Ω_S and $\partial\Omega$ for all $t \in [0,T]$.
By making the change of variables $x = \tilde{\mathbf{X}}(t,p)$ in the whole $(0,T) \times \Omega$, we reduce (3) to the weak formulation of an parabolic-elliptic problem with variable coefficients for the function $\hat{A}(t,p) := A(t, \tilde{\mathbf{X}}(t,p))$, but now the parabolic subdomain is $\hat{\Omega}$, hence independent of time.
In a further step we introduce an Steklov-Poincaré (that is, Dirichlet to Neumann) operator associated to the elliptic problem which $\hat{A}(t)$ satisfies in $\Omega^e := \Omega \backslash \overline{\hat{\Omega}}$. This enables to reduce further the problem to a non-degenerate parabolic problem posed in $(0,T) \times \hat{\Omega}$ with a non-local boundary condition involving the Steklov-Poincaré operator. This parabolic problem is a special case of an abstract parabolic problem of the form

$$\begin{cases} \text{Find } \ u \in L^2(0;T;V) \cap H^1(0;T;H) \quad \text{s.t.} \\ B(t)\frac{du}{dt} + E(t)u + C(t) = F(t) \\ u(0) = u_0, \end{cases} \tag{4}$$

where V and H are Hilbert spaces satisfying the usual assumptions (see, for instance, [3]), $B(t) \in \mathcal{L}(H)$, $E(t) \in \mathcal{L}(V;V')$, $C(t) \in \mathcal{L}(V;H)$, $F \in H^1(0,T;V')$ and $u_0 \in V$. By deriving a priori estimates, we obtain regularity results for (4) in terms of the smoothness of the operator coefficients, which generalize classical results ([3]).
Next, we apply these results to our concrete problem. In this case the operator $E(t)$ involves the Steklov-Poincaré operator and hence an elliptic problem with time-dependent coefficients.
In this way we obtain regularity results for the restriction of $\hat{A}$ to $(0,T) \times \hat{\Omega}$. Then, by analyzing the elliptic problem which $\hat{A}(t)$ satisfies in Ω^e, we obtain regularity results for the restriction of $\hat{A}$ to $(0,T) \times \hat{\Omega}^e$.
Finally, we use again the change of variables $x = \tilde{\mathbf{X}}(t,p)$ to obtain the regularity of A from that of $\hat{A}$.

Keywords: Parabolic-elliptic problem, moving parabolic subdomain, Lagrangian

MSC 2010: 35K20, 35K65

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6th Iberian Mathematical Meeting
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A local ADER FV/FE projection method for Navier-Stokes equations with turbulence and species transport

A. Bermúdez[1], **S. Busto**[1], **J.L. Ferrín**[1], **L. Saavedra**[2], **E.F. Toro**[3], **M.E. Vázquez-Cendón**[1]

The scope of this talk is to present a second order in space and time finite volume/finite element projection method to solve the Navier-Stokes equations. Moreover, transport of species, energy conservation laws and a $k-\varepsilon$ standard model will also be considered (see [1]).

Starting with a 3D tetrahedral finite element mesh of the computational domain, the momentum equation is discretized by a finite volume method associated with a dual finite volume mesh. The resolution of Navier-Stokes equations coupled with a $k-\varepsilon$ turbulence model requires the use of a high order scheme. The ADER methodology for solving advection-diffusion-reaction equations (see [3]) is extended to 3D. The developed Local ADER method profits from the dual mesh and uses the Galerkin approach to reduce the stencil and the computational cost. The obtained scheme is second order in space and time.

Concerning the projection stage, the pressure correction is computed by a piecewise linear finite element method associated with the initial tetrahedral mesh. Passing the information from one stage to the other is carefully made in order to get a stable global scheme (see [2]).

Finally, the analysis of the order of convergence by means of academic problems and several classical test problems from fluid mechanics are presented.

Keywords: Navier-Stokes, RANS $k-\varepsilon$, Finite Volumes, ADER methodology, Finite Elements

MSC 2010: 35L00, 35Q00, 65M00, 76D00, 76F06, 76M01, 76M02, 76R01, 76R02

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Viscoelastic biodegradable materials: modelling, mathematical analysis and medical applications

Paula de Oliveira[1]

Viscoelastic properties of materials represent a compromise between viscous and elastic responses, under mechanical stress. Biodegradation is the erosion of materials by the action of biological processes that cause a progressive breakdown of the material. The degradation and the unique viscoelastic properties of polymers give them a central role in controlled drug delivery to provide sustained release of therapeutic agents while avoiding removal surgery.

The release of drug is governed by an instantaneous swelling, a nonlinear diffusion, a stress driven convection and a decrease of the polymer molecular weight. The interaction of these phenomena is represented by a system of partial integro-differential equations, coupled with initial and boundary conditions. The qualitative properties of the solution are studied and the stability of the system is analyzed.

Medical applications are addressed. In vivo evolution of the concentration of drug, released from a biodegradable viscoelastic implant, is analyzed. Numerical simulations illustrate how to tune polymeric material properties that give rise to predefined release profiles.

The material presented in this talk is a joint work with J. Ferreira and P. Silva.

Keywords: Drug release, viscoelastic polymers, biodegradable implant

MSC 2010: 35K55, 65N06

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Continuation of periodic orbits in domains heated by the side

Marta Net[1], **Juan Sánchez Umbría**[1]

The usefulness and advantages of computing periodic solutions of dissipative PDEs by means of Newton-Krylov continuation methods [1, 2, 3] will be illustrated in the presentation with the application of the method to the calculation of the periodic flows arising in a tall rectangular cavity laterally heated. This problem has long been studied because of its relevance in industrial applications, for instance, in the successful growth of single liquid crystals, the design of large-scale laser systems, or the optimal heating or cooling and isolation of buildings.

It was known that in this problem there are multiple stable periodic and quasiperiodic orbits coexisting in the same range of parameters, which origin was assumed in [4] from the comparison of the critical eigenfunction of the steady solutions at the bifurcating points and the spatial and temporal structure of the periodic orbits. Here it is shown that the orbits detected previously by time evolution (and other branches calculated only by continuation techniques) arise directly from the basic steady flow without intermediate turning points or symmetry-breaking bifurcations. The Neimark-Sacker points on the branches of periodic solutions have been determined with precision for a long range of Rayleigh numbers.

Keywords: Periodic orbits, Bifurcations and Stability, Thermal convection

MSC 2010: 37G15, 76Exx

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Quasiconvexity, Rank-One Convexity and the Non-Negativeness of Polynomials

Luís Bandeira[1], **Pablo Pedregal**[2]

We report our work about non-negativeness of polynomials and the main necessary and sufficient conditions for weak lower semicontinuity of integral functionals in vector calculus of variations. The celebrated theorem about sum of squares and non-negativeness of polynomials, due by David Hilbert, plays a special role, providing new tools to investigate rank-one convexity of functions defined on 2×2-matrices. For these results, we explore some consequences and examples.
We also explore the relationship between quasiconvexity and non-negativeness of certain polynomials, in particular, the case where the integrand of an integral functional is a fourth-degree homogeneous polynomial.

Keywords: Quasiconvexity, Rank-one Convexity, Polynomials, Sums of Squares

MSC 2010: 49J45, 49J10

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On the minimization of the urban heat island effect in metropolitan areas

Lino J. Alvarez-Vázquez[1], **Francisco Fernández**[2], **Aurea Martínez**[1], **Miguel E. Vázquez-Méndez**[3]

In this work we combine optimization techniques, numerical simulation and optimal control theory of partial differential equations in order to mitigate the *urban heat island* (UHI) effect: a very usual environmental phenomenon where the metropolitan areas present a significantly warmer temperature than their surrounding areas, mainly due to the consequences of human activities. The temperature difference between urban areas and the surrounding suburban or rural areas can reach upto 5 degrees. These temperature differences are larger at night than during the day, and is strongly marked when winds are very weak. At the present time, UHI is considered as one of the major environmental problems in the 21st century as an undesired result of urbanization and industrialization of human civilization [1]. Mitigation of the UHI effect can be accomplished through the use of green roofs or of lighter-coloured surfaces in urban areas, or - as will be addressed in this study - through the setting of new green zones inside the city. So, we introduce a well-posed mathematical formulation of the environmental problem (related to the optimal location of green zones in metropolitan areas), we propose a numerical algorithm for its resolution, and finally we present several numerical results.

Keywords: Partial differential equations, Optimal control, Urban heat islands

MSC 2010: 35B37, 49J20

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Second-order structured deformations: relaxation, integral representation and applications

Ana Cristina Barroso[1], **José Matias**[2], **Marco Morandotti**[3], **David R. Owen**[4]

Second-order structured deformations of continua provide an extension of the multiscale geometry of first-order structured deformations by taking into account the effects of submacroscopic bending and curving. In this talk an integral representation for a relaxed energy functional in the setting of second-order structured deformations is presented. Our result covers inhomogeneous initial energy densities (i.e., with explicit dependence on the position in the given body). Explicit formulas for bulk relaxed energies in a particular example, as well as some applications, are also provided.

Keywords: structured deformations, relaxation, energy minimisation

MSC 2010: 49J45, 74G65, 74M25

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Optimal Design of Fractured Media with Prescribed Macroscopic Strain

José Matias[1], **Marco Morandotti**[2], **Elvira Zappalle**[3]

In this work we consider an optimal design problem for two-component fractured media for which a macroscopic strain is prescribed. Within the framework of structured deformations, we derive an integral representation for the relaxed energy functional. We start from an energy functional accounting for bulk and surface contributions coming from both constituents of the material; the relaxed energy densities, obtained via a blow-up method, are determined by a delicate interplay between the optimization of sharp interfaces and the diffusion of microcracks. This model has the far-reaching perspective to incorporate elements of plasticity in optimal design of composite media.

Keywords: Structured deformations, optimal design, relaxation

MSC 2010: 49J45, 74A60

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Modeling the physiopathology of the vascular system

Jorge Tiago[1], **Telma Guerra**[2], **Telma Silva**[3], **Adélia Sequeira**[4]

Vascular diseases, such as brain aneurysms and atherosclerosis, are the main cause of death in the western countries. Such pathologies are not fully understood and lack precise diagnosis procedures. The mathematical modeling of blood flow in the cardiovascular system, both in normal and pathological conditions, may be the way to provide a computational tool to be used for diagnosis, prognosis or training purposes. In this sense, accurate numerical simulations must be achieved, in order to be considered reliable. However, this can be a challenge since important data, needed to close the mathematical model, is usually missing. To overcome such difficulty, variational data assimilation techniques can be used. Besides, in the case of complex pathologies, such as atherosclerosis, a cascade of biochemical and biomechanical factors must be modeled. Therefore, such models results in coupled systems of partial differential equations, which are non trivial from the analysis point of view. In this talk, we will discuss some mathematical and numerical aspects related to these issues.

Keywords: Blood flow, atherosclerosis, data assimilation, numerical simulations

MSC 2010: 92C, 35Q92, 49J20

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Differential equations subject to uncertainty: some connections between different approaches

Rosana Rodríguez-López[1]

One of the approaches to include uncertainty in mathematical models is fuzzy mathematics. Fuzzy differential equations have become an intensive field of research, motivating the development of several concepts of derivatives for functions with fuzzy values. This way, the concept of Hukuhara differentiability has been completed with other generalized concepts such as strongly generalized derivative or generalized Hukuhara differentiability. Besides, other techniques have been proposed in order to deal with this type of equations, we mention, for instance, Zadeh's Extension Principle or the solvability through differential inclusions.

It is well known that equivalent real differential equations might be no longer equivalent in the fuzzy context. Indeed, using the concept of a fuzzy differential to obtain a solution, the fuzzy solutions to related linear problems that are identical in the real case may exhibit very different behavior. Being Zadeh's Extension Principle a useful approach to avoid these drawbacks but presenting also some difficulties concerning the computational treatment, we consider here the approach of solutions based on strongly generalized differentiability [1, 2, 3], and the method of differential inclusions [5], which does not require the consideration of a concept of fuzzy derivative.

In particular, we establish some connections between the expression of the solution to linear fuzzy differential equations via differential inclusions and the corresponding strongly generalized solutions. This study is based on the explicit expression of the solutions to the Cauchy problem for linear fuzzy differential equations provided in [6]. Some related works on initial value problems are [4, 8], while some results for certain types of boundary value problems following different approaches can be found, for instance, in [7, 9].

Keywords: Fuzzy differential equations, Strongly generalized differentiability, Differential inclusions' approach

MSC 2010: 34A07, 34A30, 34A60, 26E50

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First results on the modelling of heavy metals phytoremediation

Carmen Rodríguez[1], **Lino J. Alvarez-Vázquez**[2], **Aurea Martínez**[2], **Miguel E. Vázquez-Méndez**[3]

This work deals with the numerical modelling of the different processes related to the phytoremediation methods for remediation of heavy metal-contaminated environments. Phytoremediation is a cost-effective plant-based approach of remediation that takes advantage of the ability of plants to concentrate elements and compounds from the environment and to metabolize them in their tissues (toxic heavy metals and organic pollutants are the major targets for phytoremediation).

Within the framework of water pollution, biosorption (which uses the ability of biological materials to remove and accumulate heavy metals from aqueous solutions) has received considerable attention in recent years because of its advantages compared to traditional methods. Biosorption uses cheaper materials (such as naturally abundant algae and microalgae) as biosorbents. Algae possess the ability to take up toxic heavy metals from the environment, resulting in higher concentrations than those in the surrounding water.

In order to analyse this environmental problem, we propose a bidimensional mathematical model coupling the system for shallow water hydrodynamics with the system of coupled equations modelling the concentrations of heavy metals, algae and nutrients in large waterbodies. In this first mathematical approach to the problem from the viewpoint of environmental control, we present a numerical algorithm for solving the system, and several preliminary computational examples for a simple realistic case.

Keywords: Partial differential equations, Numerical modelling, Heavy metals

MSC 2010: 35B37, 49J20

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Two fractional differential equations in mechanics

Daniel Cao Labora[1]

The main goal of the talk is to make an introduction to fractional calculus and to show its usefulness when dealing with two particular physical problems.

Fractional calculus studies integrals and derivatives of non-integer orders. Some reasonable ideas of how this generalization could be made are presented. We build the Riemann-Liouville fractional integral and an analogous expression for the associated fractional derivative is found too. The ideas of this first part can be found in [3].

The previous concepts allow to formulate two fractional differential equations (Abel equation and Bagley-Torvik equation) that are deduced from two physical problems. The first problem cares about finding the "tautochrone curve" and we will follow the basic lines exposed in [1]. The second one deals with the "motion of an immersed plate" and we will use [2].

Keywords: Fractional calculus, Tautochrone curve, Motion of an immersed plate

MSC 2010: 26A33

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Modelling air pollution due to traffic flow in urban networks

Miguel E. Vázquez-Méndez[1], **Lino J. Alvarez-Vázquez**[2], **Néstor García-Chan**[3], **Aurea Martínez**[2]

Traffic flow is known as the main pollution source in urban zones. In big cities there exist several thousands of vehicles (even millions of them in megacities), so estimating the pollution emission rate due to traffic flow is a hard task.

To address this environmental issue, we propose a methodology consisting of combining the 1D Lighthill-Whitham-Richards (LWR) model with a classical 2D advection-diffusion-reaction pollution model. The pollution model uses a source term that takes into account the traffic flow contamination via a Radon measure supported on a road network in a 2D urban domain.

In this work we establish the existence of solution for the combining model, and detail a complete algorithm to solve it: using the supply-demand method for the LWR model with a characteristic Lagrange finite element method for the pollution model. Finally, some numerical experiments on a real urban domain: the Guadalajara metropolitan area (Mexico) are presented, where the final aims are addressed to the control of the air pollution.

Keywords: Partial differential equations, Numerical simulation, Traffic pollution

MSC 2010: 35B37, 49J20

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Simulación numérica en la U.P.T. de As Pontes

J.L. Ferrín[1], **A. Bermúdez**[1], **L. Saavedra**[2]

Numerosos problemas relacionados con los fluidos tienen lugar en un central térmica de carbón pulverizado como es la que la empresa Endesa tiene en As Pontes. En esta comunicación se abordará el análisis de flujo bifásico, compresible, turbulento y reactivo que tiene lugar en el interior de la caldera. Ello se ha abordado de dos formas diferentes: en primer lugar, definiendo los modelos matemáticos que describen los fenómenos físico/químicos de interés y, posteriormente, implementando los métodos numéricos que los resuelve y en segundo lugar, utilizando una herramienta de mecánica de fluidos comercial.

Así, empezaremos presentando el modelo matemático que se ha desarrollado para la combustión de carbón pulverizado (ver [1]), el cual incluye la evaporación de la humedad, la devolatilización y la gasificación del *char*, con la posibilidad de que ocurran de forma simultánea. Ese modelo se ha implementado en un código de elementos finitos, en el que se utilizan métodos numéricos análogos a los semilagrangianos y que serán presentados en otra ponencia de la sesión, y validado en [2], utilizando para ello los datos de una llama generada en un chorro de carbón pulverizado, y obteniendo resultados como el que se muestra en la Figura 1.

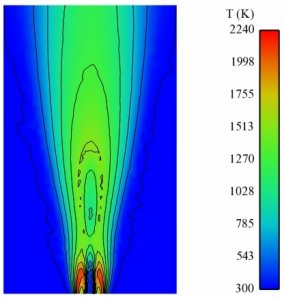

Figura 1: Temperatura de la mezcla gaseosa [K]

Por otra parte, utilizando un software comercial de Mecánica de Fluidos Computacional (CFD), se han simulado numéricamente algunos de los procesos que tienen lugar en una Central Térmica de carbón pulverizado como, por ejemplo, la organización de la combustión en el interior de la caldera y la distribución de gas y partículas de carbón en las cajas de viento (ver [3]) o en un quemador.

Keywords: pulverized coal combustion, mathematical model, CFD

MSC 2010: 80A25, 35Q35

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A numerical method for the space distributed order Riesz fractional diffusion equation

Magda Rebelo[1], **Luísa Morgado**[2]

In the recent decades a considerable and increasing attention has been devoted to fractional differential equations mainly because it has been observed that models including noninteger orders in their derivatives could describe accurately certain processes than those that restrict the orders of the derivatives to integer values.

Application problems of fractional differential equations are nowadays recognized in several areas of science and engineering, as Physics, Finance, Medicine, Biology and Biochemistry, just to name a few (see book [2]).

The Riesz fractional diffusion equation (RFDE) has been found in a broad variety of engineering, biological and physics processes. For example, for the modeling of the propagation of the electrical potential in heterogeneous cardiac tissue ([3] and [4]) and for modeling the growth and dispersion of population species ([1]).

In this work we analyse the numerical approximation of a generalization of RFDE, which may be achieved by considering the so-called distributed order equation. The Riesz fractional diffusion equation with distributed order may be written as:

$$\frac{\partial u(x,t)}{\partial t} = \int_1^2 c(\alpha) \frac{\partial^\alpha u(x,t)}{\delta |x|^\alpha} d\alpha + f(x,t,u(x,t)), \;\; 0 < t \leq T, \; 0 < x < L,$$

where $c(\alpha)$ acts as a weight for the order of differentiation.

We present a first order (time and space) accurate implicit scheme for the numerical approximation of the distributed order RFDE with appropriate boundary and initial conditions. The unconditional stability and the convergence order of the numerical scheme are analysed and illustrated through some numerical examples.

Keywords: Fractional differential equation, Riesz derivative, diffusion equation, implicit finite difference method, stability

MSC 2010: 35R11, 65M06, 65M12

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Numerical solution of some Fokker-Planck kinetic equations

Óscar López Pouso[1]

The talk will be focused on the numerical solution of the Fokker-Planck equations

$$\mu\frac{\partial\psi}{\partial z}+\alpha\psi=\sigma\frac{\partial}{\partial\mu}\left[\left(1-\mu^2\right)\frac{\partial\psi}{\partial\mu}\right]+W \tag{1}$$

and

$$\mu\frac{\partial\psi}{\partial z}+\alpha\psi=\sigma\left\{\frac{\partial}{\partial\mu}\left[\left(1-\mu^2\right)\frac{\partial\psi}{\partial\mu}\right]+\frac{1}{1-\mu^2}\frac{\partial^2\psi}{\partial\theta^2}\right\}+W \tag{2}$$

accompanied by appropriate closing conditions. See [1] and references therein.

In Equations (1) and (2), $\psi=\psi(z,\mu,\theta)$ is the angular flux density of charged particles, and $\alpha\geq 0$, $\sigma>0$, and W are given functions. Also, $(z,\mu,\theta)\in[Z_{\rm ini},Z_{\rm fin}]\times[-1,1]\times[0,2\pi)$.

The solution to the Fokker-Planck equation is, under some circumstances, close to the solution to the Boltzmann equation, which may be much more difficult to solve.

Keywords: Fokker-Planck equation, finite differences, Crank-Nicolson scheme, two-way diffusion equations, continuous scattering operator

MSC 2010: 35Q84, 65-05, 65Z05, 78M20, 78A35, 35K65

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S3
Theoretical Informatics/Computer Science

In our daily life we often interact with computing devices in an explicit form (as on using a telephone or a cash dispenser) or inadvertently (for example, with the embedded processors in a car); we live this way surrounded by devices that manage our environment and even decide for us. Though normally it remains hidden for the usual user, the functioning of these devices is based on numerous concepts developed and studied by the theoretical computer science, in which mathematical and computational aspects converge.

According to SIGACT (Special Interest Group on Algorithms and Computation Theory of the ACM), the field of the computation sciences (or theoretical computer science) includes the study of algorithms, data structures, theory of the computational complexity, distributed computation, parallel computation, VLSI, automatic learning, computational biology, computational geometry, information theory, cryptography, quantum computation, algorithmic theory of numbers, symbolic computation, program semantics and verification, formal methods, automata theory, and randomness. Such list requires frequent revisions because the computer science is in continuous development and new models of calculation are defined at a frenzied pace, as the ubiquitous computation or the computation in the cloud.

The series of conferences provides a sample of the research realized in this field in the Iberian Peninsula.

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Algorithmic Debugging: A Road Map

Josep Silva[1]

Algorithmic debugging [1, 2, 3] is a semi-automatic technique to discover bugs in programs. It was originally defined in the logic programming paradigm, but it has been later adapted to all paradigms. The high level of abstraction of this technique allows for debugging programs without the need to see (or know about) the source code of the program being debugged. In this work I analyze the evolution of algorithmic debugging along its history, which last more than three decades. I present the milestones reached along that time, the problems found, the solutions proposed, and the main applications of algorithmic debugging. On the practical side, I analyze the main features that a modern algorithmic debugger must have, and I review the use in the industry and the academia of current algorithmic debuggers.

Keywords: Software Engineering, Debugging, Algorithmic Debugging

MSC 2010: 68N15, 68W40

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Software Development with Automatic Deductive Verifiers*

Paqui Lucio[1]

Formal verification is a technique that mathematically proves the absence of errors in computer programs. Deductive verification is based on the axiomatic semantics of programming languages that was introduced by C.A.R. Hoare in the late 60s: Hoare's formal system or Hoare's logic ([2]. Such formal system has been widely studied and applied in different areas of programming languages and software engineering, given rise to the software development method known as "design by contract", programming languages that incorporate automated assertions, and verifiers that check (or help the user to check) that these assertions are true. In particular, deductive verifiers are based on using an automated theorem prover to automatically show the properties in which the program correction (with respect to its specification) is based. In this talk we introduce the evolution from Hoare logic to today's automated deductive verification tools (see e.g. [1]).

Keywords: Formal verification, Assertion, Hoare's logic.

MSC 2010: 68N30, 68Q55

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Asymptotic Efficiency of systems under RESTART and CHECKPOINTING regimes when the sequence of tasks is of random size*

Antonio Sodre[1]

In many areas, such as computer science, wireless networks and queuing theory, tasks are subject to failures. Examples include the execution of a computer program, package transmission, the copy of a file from a remote location using a protocol as FTP or HTTP, channel reservation in cognitive radio networks, information retrieval models, among others (see, for example, [4, 7, 8] and [6]).

We study a sequence of tasks whose *ideal* time to completion is random and each task is subject to failures. Once a failure occurs, the task is restarted. Hence, the *actual* time to complete the task is larger than the *ideal* time. The sequence of *ideal* times to complete each task as the distance between points of a stationary point process in $\mathbb{R}$. Asymptotic efficiency is the limit of the ratio of the sum of *ideal* times over the sum of *actual* times.

[1] and [3] study the RESTART scheme for one random task of unbounded length. The *actual* task time may have infinite moments and it is heavy-tailed, a feature that spanned more realistic models of computer architecture/networks design. We extend this framework to a sequence of random tasks. We then study asymptotic efficiency of two schemes:

1. **RESTART**: Each task on the sequence is considered separately. We prove the existence of the asymptotic efficiency as the ratio of the expectation of the typical *ideal* time over the typical expectation of the typical *actual* time. *We do not rely on the distributional independence of the tasks* and several models are considered as special cases, such as, renewal processes, Markov renewal process and random walks (when there is an independent chance of a task to be redone). Optimal adjustment of distributional parameters are also presented.

2. **CHECKPOINTING**: Consider an a.s. infinite task in which the points of the point process constitutes checkpoints, i.e., while a failure does not occur, time is used to complete the subsequent tasks. Recent results on dynamics on point process (as in [5]), allow us to prove the existence of the asymptotic efficiency for renewal processes. This is a generalization of the CHECKPOINTING models in which one a.s. finite task is split into checkpoints, as in [2].

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Keywords: Mathematical problems of computer architecture, Point process, Ergodic theory on groups

MSC 2010: 68M07, 60G55, 22D40

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Toward a mathematical model for parallel topological computation within 3D digital image context

F. Diaz del Rio[1], **D. Onchis-Moaca**[2], **P. Real**[3]

This talk is concerned with the problem of developing a topologically-consistent framework for efficient parallel topological analysis and recognition in 3D digital imagery. A topological consistency proof of such systems is provided in most of the cases by means of a mathematical model of digital images and objects, under which all theoretical formulae related to topology are true and there is no room for paradoxes. The idea is to suitably generalize to 3D the promising parallel algorithmic results on combinatorial optimization over 2D digital images obtained in [1, 2, 3]. We propose a suitable generalization of the classical notion of Abstract Cell Complex, called primal-dual abstract cell complex (or pACC for short), as theoretical model of our framework and particular asymmetric pACCs, called Homological Spanning Forest (HSF, for short), as a peculiar system of "interaction dynamics", topologically describing digital objects with 6-adjacency. We aim to achieve parallel architectures compatible with this framework and to reduce drastically the time complexity in topological computations within 3D digital imagery. The new notion of topological hole tree structure of a binary 3D digital image is defined and an algorithm for computing it starting from a HSF representation is given.

Keywords: digital image, topological representation, parallel algorithm

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Fuzzy Logic Programming and the FLOPER Environment*

Ginés Moreno[1]

The challenging research area of *Fuzzy Logic Programming*, which is devoted to introduce *fuzzy logic* concepts into *logic programming* in order to explicitly deal with vagueness in a natural way, has provided an extensive variety of Prolog dialects along the last three decades. FASILL (acronym of *"Fuzzy Aggregators and Similarity Into a Logic Language"*) is a fuzzy logic language with truth degree annotations, a great variety of connectives and unification by similarity. Here we describe its syntax, operational semantics (where the fuzzified resolution principle replaces syntactic unification by weak, similarity-based unification) and declarative semantics (based on a fuzzy variant of the classical notion of least Herbrand model coping now with truth degrees and similarity relations). We also give some implementation details on the FLOPER system developed in our research group, which has been used for coding real-world applications in emergent fields like cloud computing or the semantic web.

Keywords: Fuzzy Logic Programming, Fuzzy Logic, Logic Programming

MSC 2010: 03B52, 68N17, 68T35, 06B23

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A Gentle Introduction to Linear Temporal Logic and How To Increase its Expressive Power

César Sánchez[1]

Temporal logic was invented by the philosopher Arthur Prior and developed by Hans Kamp [1] in the 1950s and 1960s to model the reasoning and representation of time. Later, in 1977 Amir Pnueli [3] introduced temporal logics to computer science in 1977 to express behaviors of computational system.

In this talk we will gently introduce linear-temporal logic (LTL), where time is interpreted as a linear infinite sequence of instants. Many results since the 80s and 90s have established connections between temporal logic, automata theory and game theory. For example, it is well known that LTL is strictly less expressive than non-deterministic finite automata on infinite words (known as Büchi automata), which has slowed the adoption of LTL to practical applications like hardware verification. Several adaptations have been proposed to mitigate this lack of expressiveness, like using fix-point calculi or encoding automata in the specification language, but most of these efforts do not preserve the elegance of a simple logic with a finite number of modal operators. At the end of this talk I will present our proposal, regular linear temporal logic (RLTL) [2, 4], to mitigate this lack of expressiveness.

Keywords: Logic, Temporal Logic, Verification, Formal Methods, Reactive Systems, Computer Science

MSC 2010: 68Q60, 68N30, 03B44

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Analysis of heart rate variability with RHRV

Xosé A. Vila[1]

Our heart beats rhythmically pushing blood throughout our body. Cardiologists use the electrocardiogram (ECG) to check if the rhythm of the heartbeat and its morphology is normal. The heart is not beating like a metronome, in fact analyzing beat-to-beat distances provide relevant informaction for the diagnosis and monitoring of patients. Mathematics and technical advances have allowed in recent years to expand the range of possible uses of this information.

This paper will explain how the heart rate variability (HRV) is obtained, which techniques are used to extract relevant information from a clinical point of view, which problems arises and how mathematics help to solve them.

Currently there are not many software tools available to clinicians able to perform automatic HRV analysis. In our group we have developed a free package for R, which called RHRV, that permitits to obtain the heart rate from the ECG and to obtain information from them.

Keywords: ECG, HRV, digital signal processing

MSC 2010: 68N99, 62P10

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Model Checking: A Formal Verification Technique with Industrial Applications

María del Mar Gallardo[1]

Nowadays, an increasing number of devices and tools include so-called critical software, that is, software that carries out highly sensitive tasks whose failure is unacceptable for security reasons. Examples of such critical applications may be found in different domains such as automotive [7], health [11], railways [9] or avionics [2]. The analysis/verification of critical systems wrt the most essential properties involves having to deal with their inherent complexity deriving from different sources. For instance, critical code is usually composed of thousands of code lines which makes a non-automatized analysis impossible. In addition, its behaviour is generally non-deterministic as critical software is usually concurrent and/or because the interaction with the environment does not occur in an orderly fashion.

In this communication, we briefly describe the foundations and current applications of model checking for the verification of critical concurrent systems. Model checking [1, 5] is a well-established formal technique characterised by displaying a good balance between mathematical rigor, needed to guarantee software correctness, and the practical applicability which comes from its algorithmic nature. Since its very beginnings, the development of the technique has been carried out in parallel with the construction of tools, the so-called model checkers.

As first glance, the idea behind model checking is simple. Roughly, to analyse a system, we only have to build the whole reachability graph produced by the system, containing all possible system behaviours, and check whether all possible executions satisfy a set of desirable properties. It is clear that the drawback to this apparently brute force technique is the well-known state explosion problem that occurs when the system graph to be analysed is too big. However, despite this non-trivial problem, the advantage of the method with respect to the deductive approaches is that it is completely algorithmic. Observe that the word algorithm in this sentence refers not only to the task of constructing the system graph, but also to the way of proving properties on this graph. This is the strength of the model checking technique and the reason why E.M. Clarke and E.A. Emerson, and J. Sifakis received the ACM Turing Award in 2007 [4]. The two research groups independently found model checking algorithms to verify CTL (computational tree logic) properties on systems [3, 10]. The complexity of these algorithms is polynomial. Specifically, the model checking procedure in [3] ran in a time proportional to the square of the system size (the number of reachable states) and linear to the size of the property analysed. However, these results were clearly improved upon in later papers. These model checking

algorithms were based on the iterative fixpoint calculations of basic temporal modalities. An interesting result from M. Vardi and P. Wolper [12] was the observation that when desirable properties are described in LTL (Linear Temporal Logic), systems to be analysed and properties are formally equivalent under the common notion of automata. Thus, combining software systems and properties is reduced to constructing a product automata, which can be done automatically and efficiently. This new interpretation of model checking made it possible to construct very efficient model checkers such as SPIN [6] developed by G. J. Holzmann who received the ACM Software System Award in 2001.

Over the last 25 years, many model checking algorithms and associated techniques have appeared, most of them trying to palliate the state explosion problem (symbolic model checking, abstract model checking, symmetry reduction algorithms, partial order reduction algorithms and so on). Currently, we can say that although the technique has a physical limitation due to the state space problem, in practice, it can be successfully applied to real systems. For instance, the symbolic model checker NuMSVM has been able to analyse systems with 10^{120} reachable states [8].

Therefore, we can conclude that model checking is a 'push-button' verification technique. Thus, a model checker accepts the specification of a system and a property written in temporal logic and returns 'yes' when all the possible executions of the system satisfy the property, or 'no', when the tool finds a malfunctioning execution. In addition, in the latter case, the tool also returns the erroneous behaviour (the counterexample) which can be used to debug the system. The fact that the model checkers appear as software tools whose inner complexity is hidden from users and that can be used by non-expert programmers has led to the gradual integration of model checking, and formal methods, in general, in the industry. Currently, many software companies such as Lucent Technologies (currently, part of NOKIA), Intel, NASA, Microsoft and IBM have departments dedicated to software analysis using model checking.

Keywords: Critical Software, Verification, Model Checking

MSC 2010: 68-02

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6th Iberian Mathematical Meeting
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Efficient Computation of Absolutely Normal Numbers

Jack H. Lutz[1], **Elvira Mayordomo**[2]

A real number x is *absolutely normal* if, for every base $b \geq 2$, every two equally long strings of digits appear with equal asymptotic frequency in the base-b expansion of x. We discuss recent progress on efficient algorithms for computing real numbers that are absolutely normal.

Keywords: absolutely normal numbers, finite-state randomness, Lempel-Ziv algorithm, martingale diagonalization, nearly linear time

MSC 2010: 03D32, 68W01, 11-04

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On the NP-Hardness of Optimizing Binarizations of Context-Free Grammars

Carlos Gómez-Rodríguez[1]

Binarization, the process of transforming a grammar to an equivalent form where each rule has at most two symbols in its right-hand side, is a key task for a wide range of parsers using context-free grammar and other grammatical formalisms. As non-trivial grammars can be binarized in multiple ways, it is convenient for efficiency reasons to choose a binarization that is as small as possible. This optimization problem has been addressed with heuristics that yield relatively compact binarizations, but no efficient algorithm is known that guarantees to generate a grammar with minimum size. In this talk, I will show that the problem of finding a minimum binarization for a given context-free grammar is NP-hard, by reduction from vertex cover. This result has been published in [1]. The result also generalizes to other, more powerful grammar formalisms.

Keywords: Complexity, binarization, grammars, context-free grammar, parsing, natural language processing

MSC 2010: 68Q17, 68Q25, 68Q42, 68T50

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A refined algorithm for testing the Leibniz n-algebra structure

J. M. Casas[1], **M. A. Insua**[1], **M. Ladra**[2], **S. Ladra**[3]

We present a refinement of the algorithm given in [2] that checks if a multiplication table corresponds to a Leibniz n-algebra structure. This algorithm is based on the computation of a Gröbner basis of the ideal which is used in the construction of the universal enveloping algebra of a Leibniz algebra and it is implemented in a Mathematica notebook by means of the NCAlgebra package.

Essentially, the refinement consists of removing all the superfluous information in the generators of the ideal; this deletion allow us to decrease highly the computation time.

A comparative analysis between both implementations is provided.

Keywords: Leibniz n-algebras, Universal enveloping algebras, Gröbner Bases

MSC 2010: 17A32, 68U99

Introduction

A Leibniz n-algebra [3] is a $\mathbb{K}$-vector space $\mathcal{L}$ endowed with an n-linear map $[-,\dots,-]\colon \mathcal{L}^{\otimes n} \to \mathcal{L}$ satisfying the Fundamental Identity

$$\big[[x_1,\dots,x_n],y_1,\dots,y_{n-1}\big] = \sum_{i=1}^{n} \big[x_1,\dots,x_{i-1},[x_i,y_1,\dots,y_{n-1}],x_{i+1},\dots,x_n\big] \tag{FI}$$

for all $x_1,\dots,x_n,y_1,\dots,y_{n-1} \in \mathcal{L}$.

When the n-bracket is skew-symmetric, the structure is named Lie n-algebra. Lie (respectively, Leibniz) 2-algebras are exactly Lie (respectively, Leibniz) algebras.

For finite-dimensional Leibniz n-algebras with basis $\{e_1,\dots,e_d\}$, the n-ary bracket is determined by structure constants $c^k_{i_1,i_2,\dots,i_n}$ such that $[e_{i_1},e_{i_2},\dots,e_{i_n}] = \sum_{k=1}^{d} c^k_{i_1,i_2,\dots,i_n} e_k$.

The problem of identify a Leibniz n-algebra structure in a given n-ary bracket is the subject of the paper [2], where a computer program in Mathematica that checks if a multiplication table satisfies (FI) is implemented. The algorithm is based on the computation of a Gröbner basis of the ideal which appears in the construction of the universal enveloping algebra of a Leibniz algebra [5], by means of the NCAlgebra

package [4] which enables the computation of Gröbner bases in non commutative associative algebras. This Gröbner basis provides a criterion in terms of existence of polynomials of degree 1 over convenient variables, which guarantees that the multiplication table corresponds to a Leibniz n-algebra or not. To decide whether a Leibniz n-algebra is a n-Lie algebra or not, it is necessary to check whether certain type of polynomials are equal to zero.

Although the implementation of this algorithm does not provide efficient results concerning times of computation in an Intel(R) Core(TM) i7-3770 CPU @ 3.40 GHz, 16 GB RAM, running Windows 7 (64 bits) with Mathematica®10, our goal in this talk is to present a refinement of the computer program in order to reduce times of computation and compare its efficiency with respect to the initial one given in [2].

Refinement of the algorithm

While we were developing the initial algorithm, it was very clear for us from the beginning [2], that the process could be quicker but the goal was not to get an efficient algorithm, at least at that point of the research. The main motivation was to proof that the Leibniz checking process can be done using Gröbner Basis. Doing things like this, we have enriched the problem and so it is possible to manage the situation from a different and useful point of view (Gröbner Basis Theory and Ideal Theory). Once the theoretical background is established, our interest changed from the existence to the efficiency of the algorithm.

The main idea, we followed to reduce computation time, was to avoid unnecessary computations and to remove all the superfluous information which is contained in the ideal.

The first criterion, we followed to avoid unnecessary computations, is the following: if we examine the proof of [2, Proposition 3.6], it is possible to check that the set of the expressions $[e_i, g_t(e_1, \ldots, e_d)]$ ($g_t \in \mathcal{D}_n(\mathcal{L})^{\mathrm{ann}}$) have an important role. If one of these brackets is not equal to zero, then the structure cannot be a Leibniz n-algebra (as if $\mathcal{L}$ is a Leibniz n-algebra, then $[-, (\mathcal{D}_n(\mathcal{L}))^{\mathrm{ann}}] = 0$).

The second criterion, we followed to remove all the superfluous information, is the following: if all the previous expressions are equal to zero, then it is easy to check, again following the proof of [2, Proposition 3.6], that it is possible to gather the information we need from a subideal of $\Phi(I)$, this subideal is $\langle \{x_i \cdot x_j - x_j \cdot x_i - \Phi(r_{[e_i,e_j]})\}_{i,j\in\{1,\ldots,m\},i<j} \bigcup \{y_i \cdot x_j - x_j \cdot y_i - \Phi(l_{[e_i,e_j]})\}_{i\in\{1,\ldots,d\},j\in\{1,\ldots,m\}} \rangle$, $m = \dim(\mathcal{D}_n(\mathcal{L})^{\mathrm{ann}})$. A natural question arises at this point, is the Gröbner Basis of this subideal finite?, the answer is affirmative because all the reductions drive us to 0 or a polynomials of degree 1.

Using these two criteria and other minor computational aspects, such that, stop the computation process as soon as we obtain the information we need, that is, no more computations will be done when the information is reached, helped us to construct a more efficient algorithm.

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The Geometry of Musical Chords according to D. Tymoczko

Ana Pereira do Vale[1]

Theoretical models for composition and musical analysis have used in recent years, mathematically structured models that gain particular importance in computational issues related to music. These models have not been started recently. The first (tonnetz) two-dimensional latice of sounds known was presented in 1739 by Leonhard Euler ([1]) This latice established a link between the notes that formed musical intervals of major third and perfect fifth. In the mid-nineteenth century the "tonnetz" were rediscovered by some music theorists, namely Arthur von Oettingen (1836–1920) ([2]) and Hugo Riemann (1849–1919) ([3]). Hugo Riemann wrote several books on musical analysis and revolutionized the theory of musical analysis so deeply, that modern theories are called Neo-Riemannian.

In Neo-Riemannian theories there is a great effort to systematize, not only through latices ("tonnetz"), but also through the definition of specific functions examples of these can be found in the works of Allen Forte (1926–2014) ([4]) or David Lewin (1933–2003) ([5]).

One objective of these approaches is to establish a music structure that is global. That could be applied in the musical analysis of different types of music:tonal music, twelve-tone music, etc. Nowadays one uses different systems for each of them.

It should be noted that these models are not intended only for the analysis of musical compositions, they also serve as a system of rules for composition. Indeed the portuguese composer Paulo Bastos who, in his master's thesis examined the "Six piano pieces" Opus 19 by Schönberg using the theory of attractive notes of Edmond Costère (([6]), has recently composed several works using this structure as a support.

Some of these mathematical models developed are geometrical. It is the case of the Theory of Chord Dmitri Tymocko ([7]) and The Geometry of Rhythms ([8]).

It is known by musicians of the 12 notes model description distributing points in a circle or a dodecahedron. Notes are represented in equivalence classes. The octave is not important. All the notes that correspond to C (treble or bass) are represented by the same point and we obtain a representation of the 12 notes as a quotient set. Tymoczko describes a similar mode for musical chords, obtaining subsets of size 2, 3 and 4, as the chords consist of 2, 3 or 4 notes. Choosing an appropriate geometric representation of these spaces, we obtain a spatial organization of the chords we will provide important data for the theory of music. It will be explained in detail the model of two-note chords and explained how it will appear the model of three-note chords.

Keywords: Music Theory, Klein bottle, Voice leading, Tonnetz

MSC 2010: 00A65, 57M60

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Multi-armed Bandits for Information Retrieval Evaluation

David E. Losada[1]

Evaluation is crucial to making progress in building better search engines. In the field of Information Retrieval, it is standard practice to build test collections and define evaluation measures to assess the effectiveness of search systems [1]. Each benchmark or test collection comprises a set of queries, a collection of documents and a set of relevance judgements. Relevance judgements are often done by humans and thus expensive to obtain. Consequently, relevance judgements are customarily incomplete. Only a subset of the collection, the pool, is judged for relevance. In popular evaluation campaigns, the pool is formed by the top retrieved documents supplied by systems participating in a certain evaluation task [3]. With multiple retrieval systems contributing to the pool, an exploration/exploitation trade-off arises naturally. Exploiting effective systems could find more relevant documents, but exploring weaker systems might also be valuable for the overall judgement process. In this talk, I will explain our research on Reinforcement Learning [4] for pooling-based evaluation of Information Retrieval systems. Our proposal is based on casting document judging as a multi-armed bandit problem [2]. This formal modelling leads to theoretically grounded adjudication strategies that improve over the state of the art. We show that simple instantiations of multi-armed bandit models are superior to all previous adjudication strategies.

Keywords: Multi-armed bandits, Information Retrieval, Evaluation, Reinforcement Learning

MSC 2010: 68P20, 68T05

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Non-degeneracy conditions in automated proving and discovery

Manuel Ladra[1], **Pilar Páez-Guillán**[1], **Tomás Recio**[2]

The automated theorem proving theory was initiated by Wu in [4], and its goal is to provide computer algebra algorithms in order to decide automatically if a given geometric statement is true. For this purpose, the method assigns coordinates to the points involved in the statement (some independent, say $x_1, \ldots, x_s$, and the others dependent on the previous ones, $x_{s+1}, \ldots, x_n$), and polynomial equations to the rest of the elements (lines, circles…) and conditions (parallelism, perpendicularity…), "translating" in this way the hypotheses (H) and the thesis (T). We symbolise the theorem as $H \Longrightarrow T$, and declare it true if $V(H) \subseteq V(T)$, roughly speaking.

We will be interested in the "non-degenerated" configurations of our theorems; i.e. those in which two different points are really different, a triangle does not collapse to a line, etc. The theorems which are true except on these exceptional cases are called *generally true*. There exist methods to determine successfully whether a theorem is generally true or not, as well as the extra polynomials needed to restrict the set of hypotheses H to the desired situation –they are called *non-degeneracy conditions*. Working over an algebraically closed field K, a theorem is generally true if and only if the ideal $(H + (T \cdot t - 1)) \cap K[x_1, \ldots, x_s]$ (where t is an auxiliary variable) is different from zero; similarly, we say that a theorem is *generally false* when $(H + T) \cap K[x_1, \ldots, x_s] \neq 0$, with the approach and terminology employed in [3]. The polynomials in the last ideal are necessary conditions for the thesis to be true: if we add them to H, and after that we obtain a generally true theorem, it is said that we have *discovered* it.

However, due to the high complexity of the algorithms to compute the previous ideals (triangularization, Gröbner basis, etc.), sometimes it is convenient to introduce manually some easy-to-guess non-degeneracy conditions, and then apply the method. At this point, a problem arises: our method is designed to deal with equations, and the non-degeneracy conditions are inequalities. The traditional way to transform a non-degeneracy condition into an equality is the so-called "Rabinowitsch trick": if we want to express $f \neq 0$, we can write $f \cdot t - 1 = 0$, and add this polynomial to H. Clearly, if $f(x_1, \ldots, x_n) = 0$, we have that $f(x_1, \ldots, x_n) \cdot t - 1 = -1$, and if $f(x_1, \ldots, x_n) \neq 0$, we take $t = 1/f(x_1, \ldots, x_n)$ and we are done. But in the context of algebraically closed fields, there is another option: the saturation $\mathtt{Sat}(H, f) = (H \colon f)^\infty = \{g \in K[x_1, \ldots, x_n] \colon g \cdot f^n \in H \text{ for some } n \in \mathbb{N}_{>0}\}$. This possibility is due to the following geometric interpretation: $V(\mathtt{Sat}(H, f)) = \overline{V(H) \setminus V(f)}$, where $\overline{U}$ denotes the Zariski closure of the set $U \subseteq K[x_1, \ldots, x_n]$.

As it is indicated in [1], both methods are related:

$$\texttt{Sat}(H, f) = (H + (f \cdot t - 1)) \cap K[x_1, \ldots, x_n]. \tag{1}$$

It is our goal to show how the choice of the method employed for introducing the non-degeneracy conditions can affect our results.

Henceforth, we will write $H_1 \coloneqq H + (f \cdot t - 1)$, $H_2 \coloneqq \texttt{Sat}(H, f)$, $\mathcal{H}_1 \coloneqq (H_1 + T) \cap K[x_1, \ldots, x_s]$ and $\mathcal{H}_2 \coloneqq (H_2 + T) \cap K[x_1, \ldots, x_s]$. By (1), we see that $H_2 \subseteq H_1$, and then $\mathcal{H}_2 \subseteq \mathcal{H}_1$: the Rabinowitsch trick provides more conditions for discovery than saturation. What is interesting is that $\mathcal{H}_1 = \texttt{Sat}(\mathcal{H}_2, f)$ (analogously, $(H_1 + (T \cdot t' - 1)) \cap K[x_1, \ldots, x_s] = \texttt{Sat}((H_2 + (T \cdot t' - 1)) \cap K[x_1, \ldots, x_s], f)$), as it is shown in [2]; so, the choice does not affect whether the theorem is generally true or false.

Anyway, we still have two different sets of additional hypotheses, $\mathcal{H}_1$ and $\mathcal{H}_2$, which we must add to the original set H to find out whether they lead to a new theorem or not. And, once again, we can introduce the non-degeneracy condition $f \neq 0$ by the traditional approach or by saturation. We differentiate four statements:

$$\begin{aligned}
St_1: &\quad H + \mathcal{H}_1 + (f \cdot t - 1) \Longrightarrow T,\\
St_2: &\quad H + \mathcal{H}_2 + (f \cdot t - 1) \Longrightarrow T,\\
St_3: &\quad \texttt{Sat}(H + \mathcal{H}_1, f) \Longrightarrow T,\\
St_4: &\quad \texttt{Sat}(H + \mathcal{H}_2, f) \Longrightarrow T,
\end{aligned}$$

being St_1 and St_4 more natural than St_2 and St_3.

Using that $\texttt{Sat}(\mathcal{H}_1, f) = \texttt{Sat}(\texttt{Sat}(\mathcal{H}_2, f), f) = \texttt{Sat}(\mathcal{H}_2, f)$, it is not difficult to prove that St_3 and St_4 are, in fact, the same statement. So, we infer from the previous results that if one of our statements is generally true, the others will be too. However, they are not all the same: using the Rabinowitsch trick, we end up with two new formally different theorems.

These results can be applied in the practical implementation of the method: if we only want to know the "class of truth" of the original theorem, or determine whether it leads to a discovery or not, it is more efficient the saturation than the Rabinowitsch trick, since the former only involves one saturation, and the latter, two. In fact, sometimes a usual computer cannot achieve this information using the Rabinowitsch trick, while no difficulties are found employing the saturation.

Keywords: Automated discovery, non-degeneracy conditions, Rabinowitsch trick, saturation

MSC 2010: 68T15, 68W30

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Posters

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The mathematical legacy of Ramón María Aller Ulloa

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In observance of the 50th anniversary of the death of Ramón María Aller Ulloa, an illustrious Galician astronomer and mathematician as well as Professor of the University of Santiago de Compostela, this document is to serve as a record of his contributions to the field of Mathematics. Without a doubt, his principal work was the book, ALGORITMIA.

However, it is also important to keep in mind his articles concerning the Parabolic Theory of Errors as well as Sets and Finite, Undefined, and Transfinite Numbers. He also wrote about some of the principals of the Theory of Sets.

Keywords: History, Mathematics

MSC 2010: 01

References

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Implementing generating functions to obtain power indices with coalition configuration

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This work is part of cooperative game theory. Let us consider the class of so-called cooperative games with transferable utility (TU-games, for short), in which agents have the opportunity to distribute the profit generated by cooperation among them. A mechanism to propose asuch distribution of the profits among the agents is called a solution or value. There is a special class of TU-games, weighted majority games, with an important role in social sciences since they can represent a Parliament or any institution where decisions are taken by vote. A solution for weighted majority games is called power index. Since the calculation of power indices can be hard in the presence of a large number of agents, different calculation methods have been proposed. Among these methods, in this setup we will use the so-called generating functions (cf. Brams and Affuso, 1976 [3] and Lucas, 1983 [4] among others). This is based in a technique of combinatorial analysis that allows enumerate the set of possible coalitions among agents, while having control about their respective weights, which it is essential for the final determination of the most important power indices. Specifically, we consider the Banzhaf–Coleman (Albizuri and Aurrekoetxea, 2006 [1]) and Owen (Albizuri, Aurrekoetxea and Zarzuelo, 2006 [2]) power indices for weighted majority games modified by a coalition configuration modeling constraints on cooperation among agents. We present calculation algorithms of them that make use of the method of generating functions. We programmed the procedure in the open language R and it is illustrated by a real life example taken from social sciences. To finish, we generalize the algorithms to a wider class of games.

Keywords: Weighted majority games, Power indices, Banzhaf–Coleman index, Shapley–Shubik index, Generating functions, Coalition configuration, Weighted multiple majority games

MSC 2010: 05A10, 91A12

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Classical genetic algorithms versus genetics algorithms with varying population size over the Wilson functions

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Genetic algorithms (GA) are a mathematical tool inspired upon the mechanisms of natural evolution and are mainly applied in the framework of function optimization. Several measures have been developed for predicting the behavior of the genetic algorithms (order, k-epistasis, ...) and several classes of "laboratory functions" have been especially designed to study their dynamics. The Wilson functions are of interest to have a good characterization of functions which are easy to optimize by genetic algorithms, particularly if the functions cause difficulties for standard optimization methods. Moreover, the size of the population is one of the most important choices that any user of Genetic Algorithms faces. This parameter have been investigated from different perspectives. Most of them consider Classical Genetic Algorithms (CGA); i.e. GA using binary codification with fixed size of population and classical genetic operators: selection, crossover and mutation. In this note we analyze CGA versus Genetic Algorithms with Varying Population Size (GAVPS) for the Wilson functions. GAVPS do not use any selection mechanism and replace it by the concept of "age" of a chromosome. By means of some experimental results, we show how the k-epistasis of Wilson functions is strongly related to their behavior.

Keywords: genetic algorithm, k-epistasis, Wilson functions

MSC 2010: 68R99

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Symbolic Computation of Drazin Inverses*

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Many authors have analyzed the properties of Drazin inverses as well as their applications. An important alternative issue in the topic is the computation of the Drazin inverse.

The problem has been approached mainly for matrices with complex numbers. Nevertheless, in a second stage, different authors have addressed the problem of computing Drazin inverses of matrices over other coefficients domains as rational function fields. Furthermore, symbolic techniques have proven to be a suitable tools for this goal. In this context, we show how to compute the Drazin inverses of matrices whose entries are elements of a finite transcendental field extension of a computable field. For this purpose, we reduce the computation of Drazin inverses over certain computable fields to the computation of Drazin inverses of matrices with rational functions as entries. As a consequence, we derive a symbolic algorithm. The algorithm is applied to matrices over the field of meromorphic functions, in several complex variables, on a connected domain and to matrices over the field of Laurent formal power series. Essentially, this algorithmic method applies symbolic computation to determine the Drazin inverse via specializations, and reduces the problem to the computation, via Gröbner bases, of Drazin inverse matrices with multivariate rational functions as entries. Furthermore, we show how to relate the specialization of the Drazin inverse of a matrix, with meromorphic function entries, and the Drazin inverse of the specialization.

More precisely, given a matrix A, the idea consists in the following three steps:

(1) [Specialization step] first we associate to A a matrix A^* whose entries are rational functions in several variables, whose entries are rational functions in several variables;

(2) [Inverse computation step] we compute the Drazin inverse of A^*;

(3) [Evaluation step] finally, from the Drazin inverse of A^*, we get the Drazin inverse of A.

The results mentioned above have been developed in the papers [1, 2].

Keywords: Drazin inverse, Gröbner bases, symbolic computation, meromorphic functions

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